Filtering in space domain

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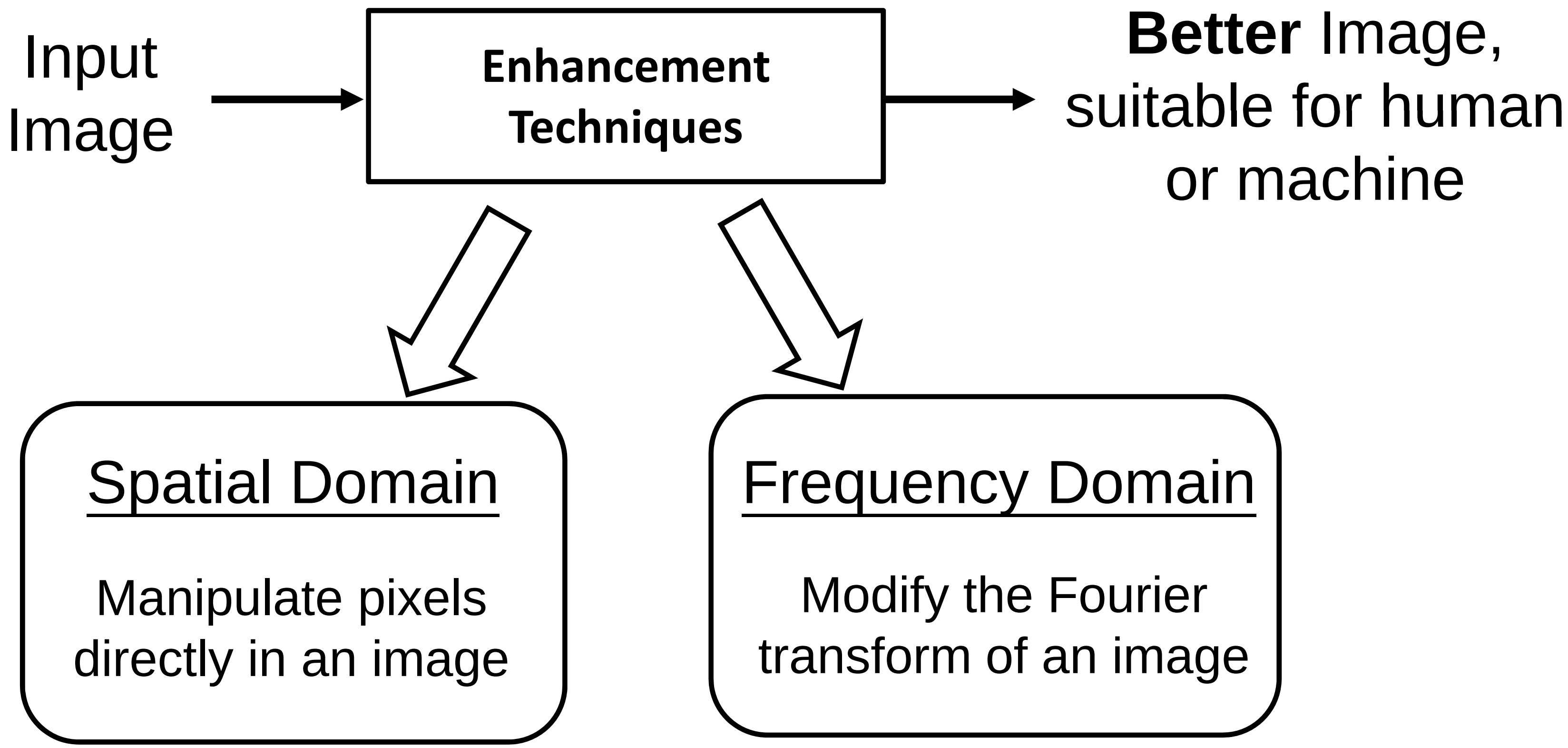
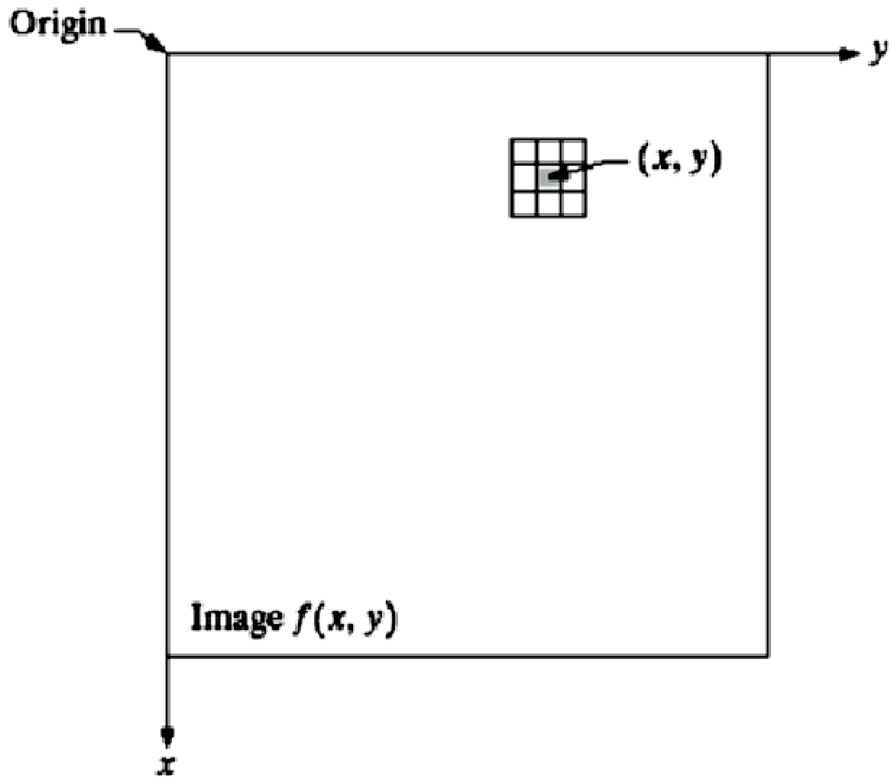
Image Enhancement



Filtering in Spatial Domain

$g(x, y) = T[f(x, y)]$



$f(x, y)$: input image

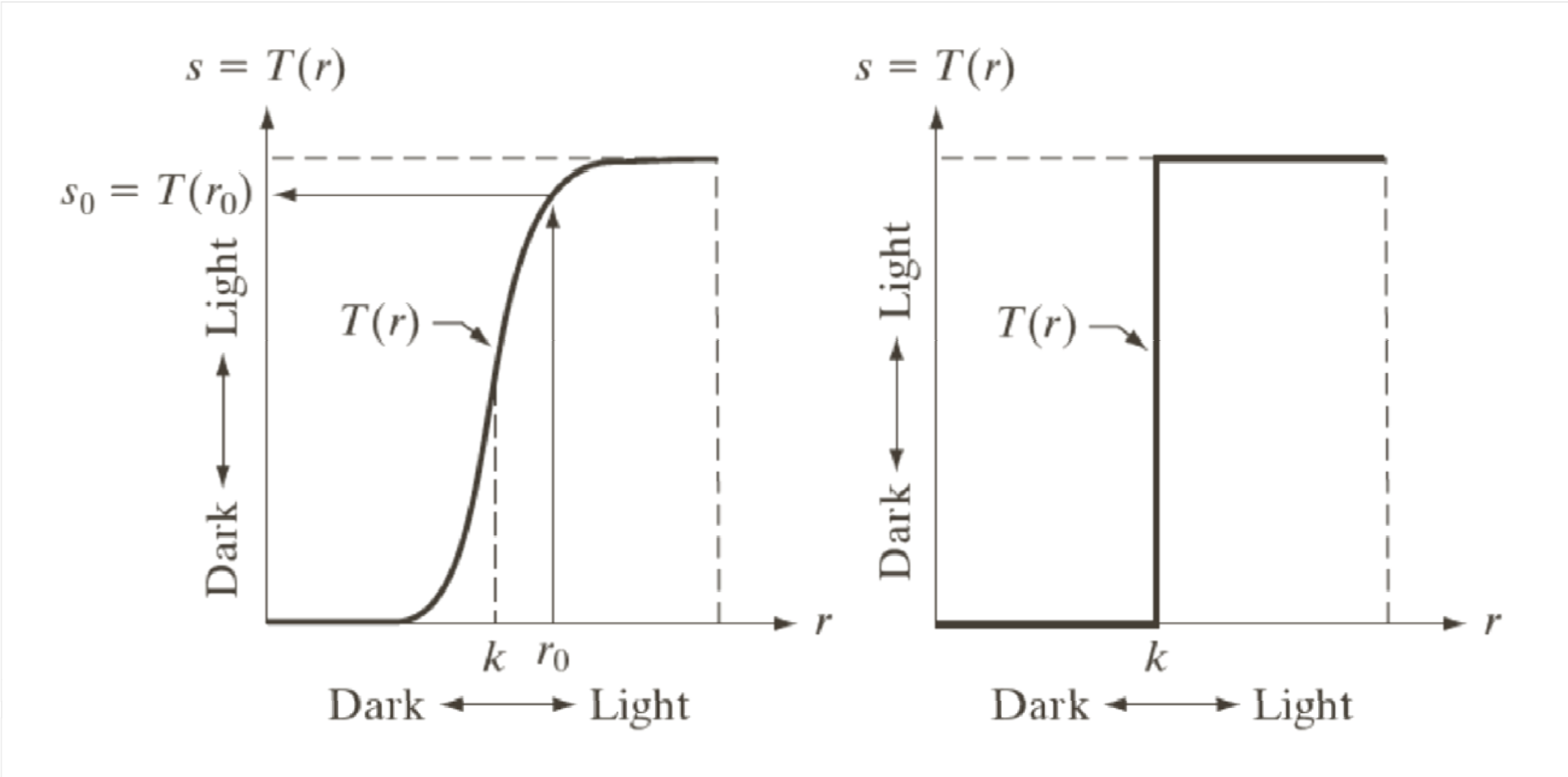
$g(x, y)$: output image

T: gray-level transformation function

Spatial filtering

- The neighborhood area predefined by T: Spatial filter, kernel, operator, mask, or window
- The smallest possible size of the kernel: 1×1
  - Intensity transformation function (T)
  - $s = T(r)$

Intensity transformation function



Contrast stretching　　　　Thresholding

Example: Contrast Stretching

Input image

Contrast stretching Thresholding

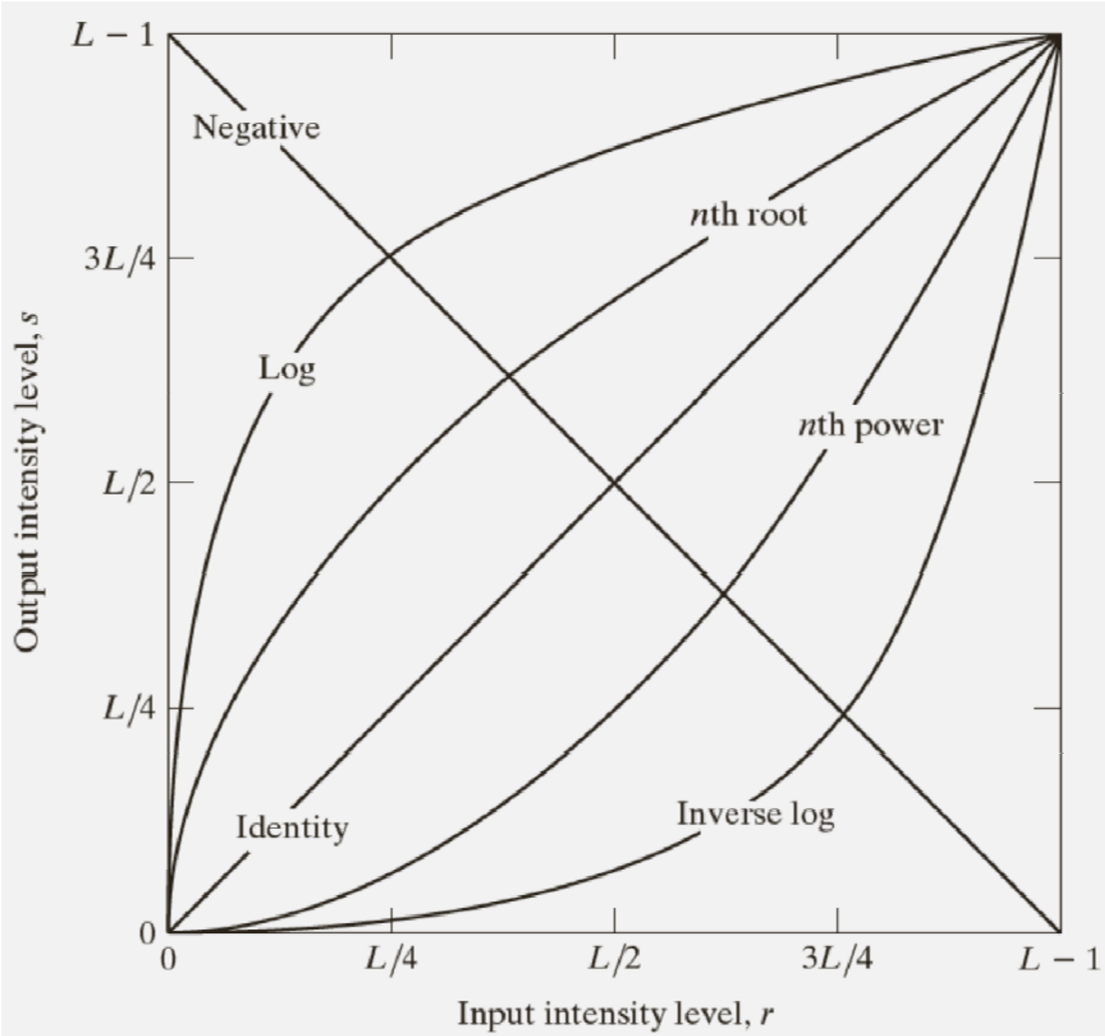
Intensity transformation function

- Power function
- Root function
- Log function

  $s = c\, log(1 + r)$

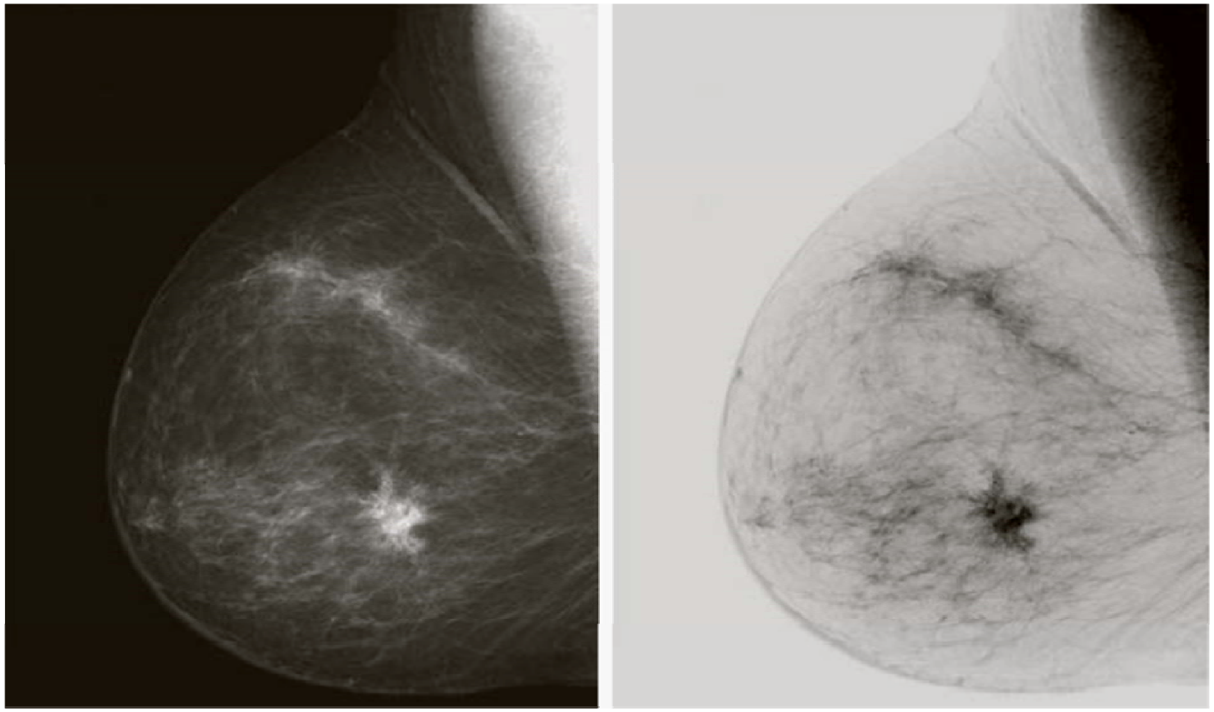
- Negative function

  $s = L-1-r$



Example: Negative Image

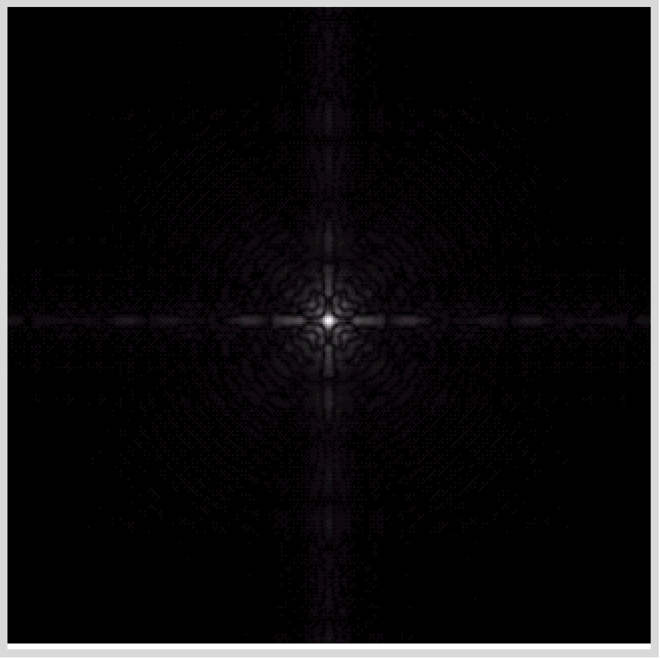
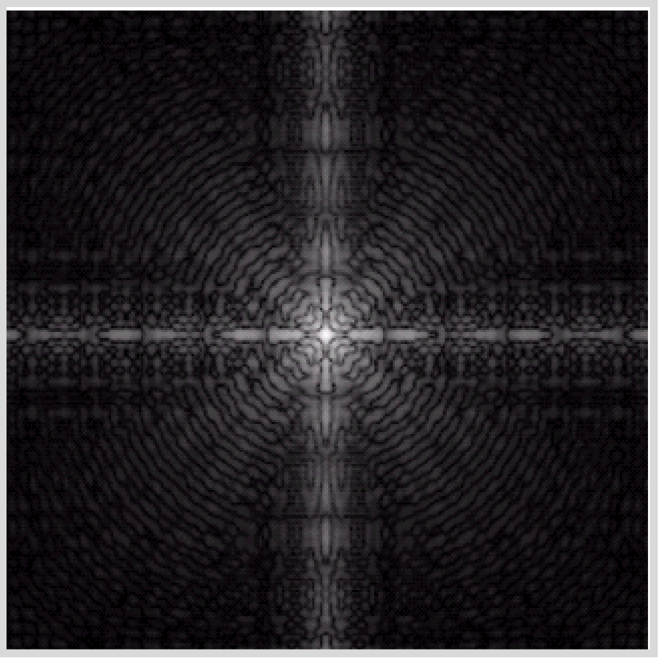
Case: X-ray Mammogram



$$g(x, y) = 255 - f(x, y)$$

Example: Log transformation

- $s = c\,log(1+|r|)$, c : scaling factor

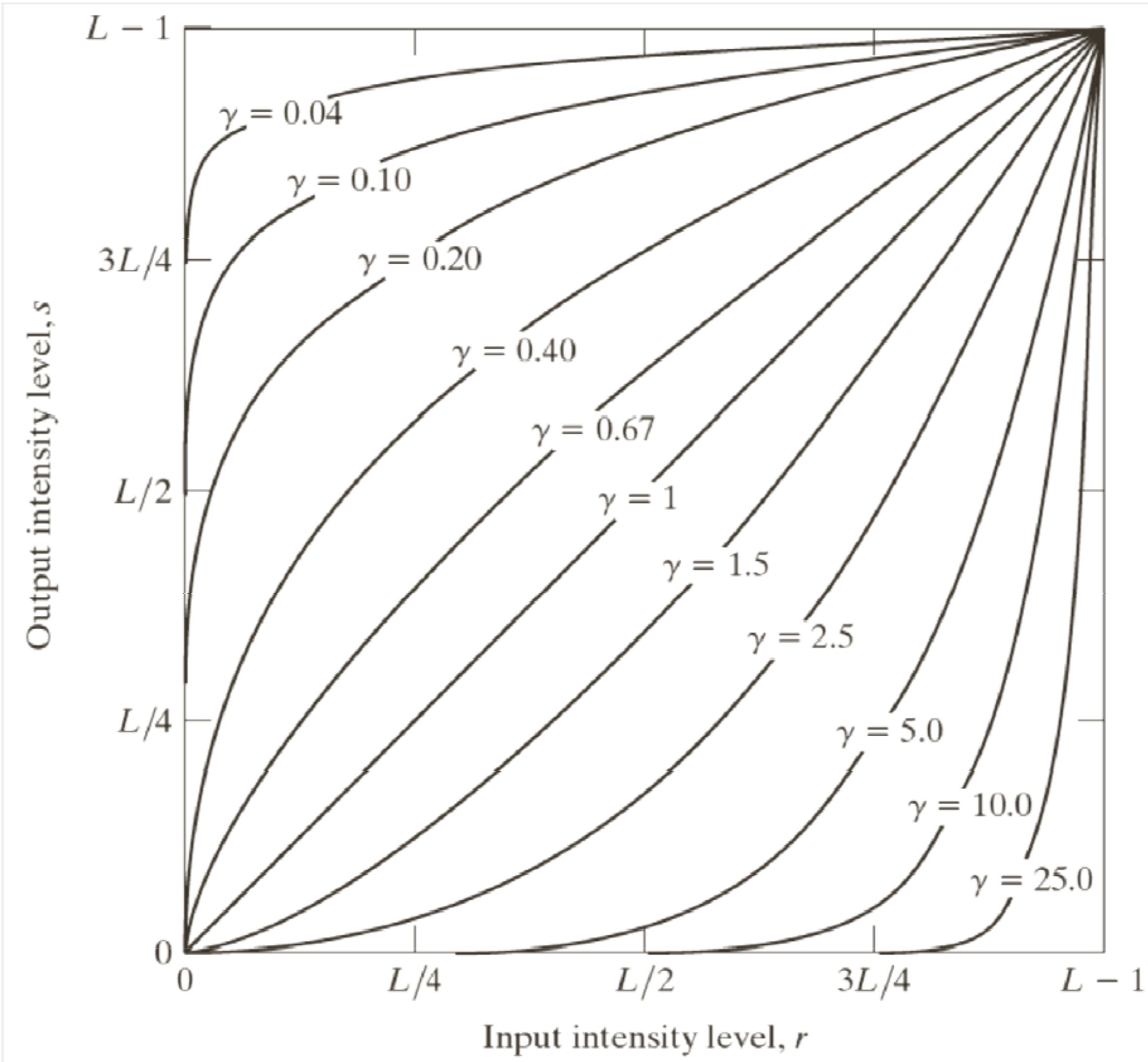



Display the 2D spatial spectrum (k-space)

Power-Law transformation

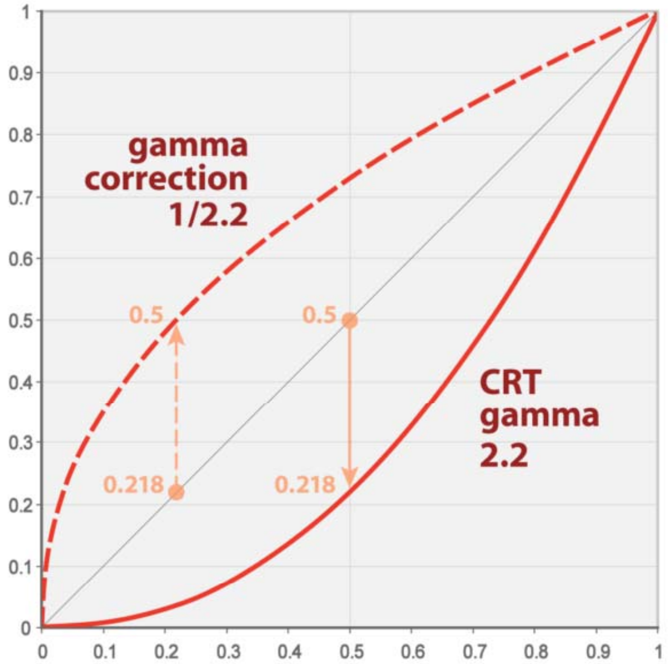
- $s = c\, r^{\gamma}$
- c, γ : positive constants
- CRT : intensity-to-voltage response follows a power function (typical value of gamma in the range 1.5-2.5)

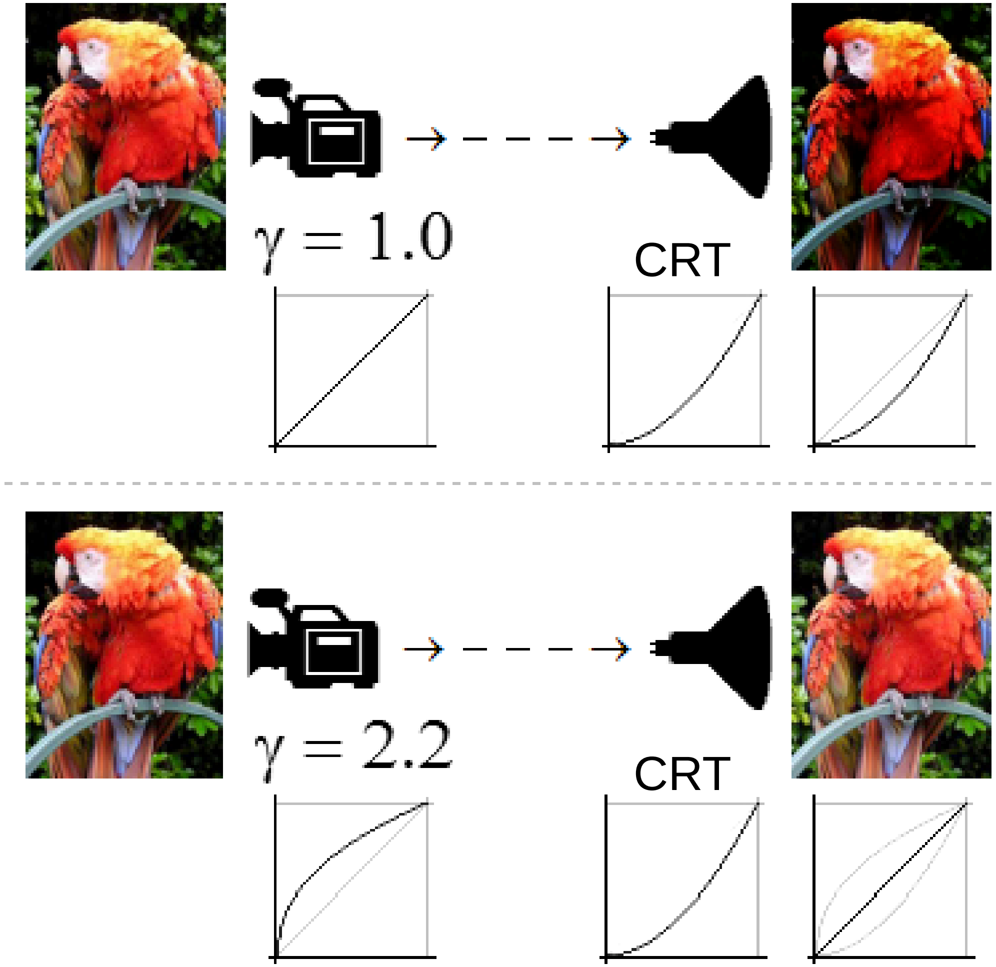
Gamma correction



$s = c r^{\gamma}$

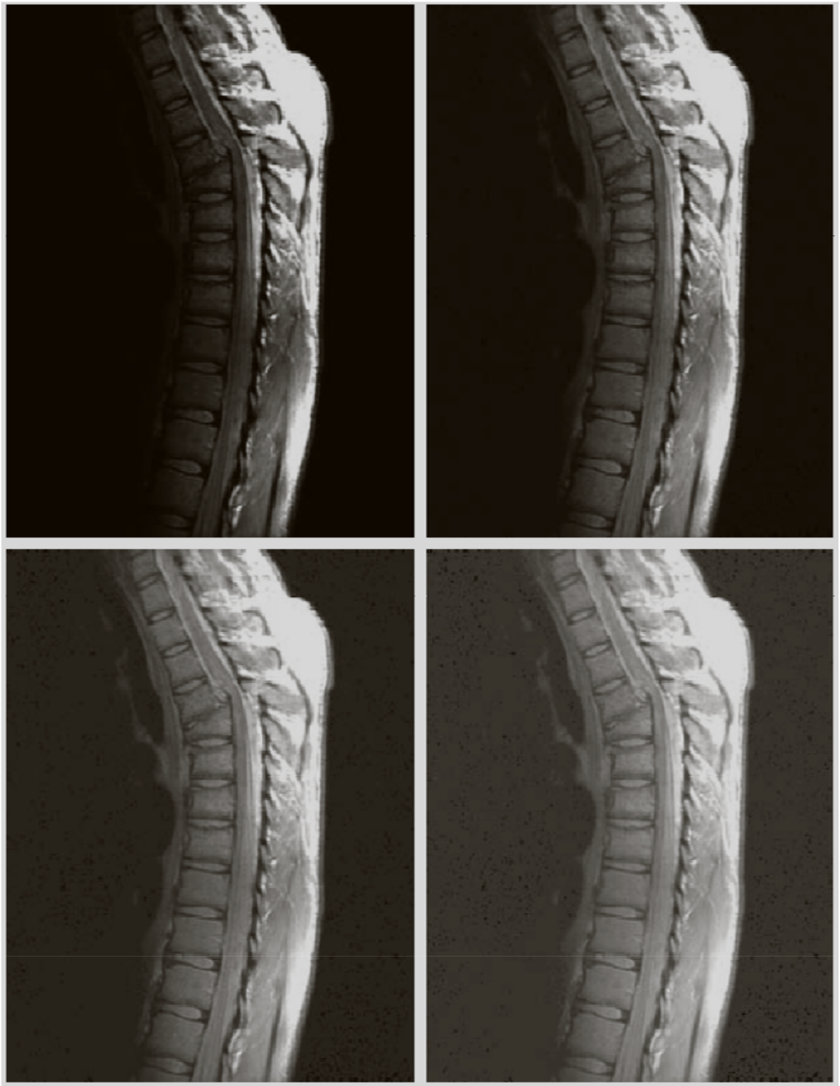
Gamma correction of CRT





Example: c = 1, gamma = ?

Case: MRI of a human spine



γ =1

γ =0.6

γ =0.4

γ =0.3

Improve visual quality

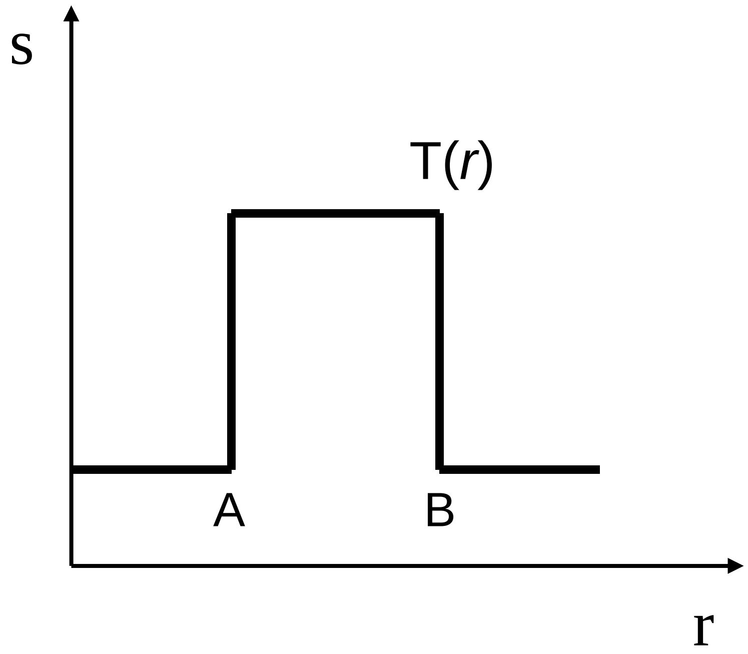
γ =1

γ =3

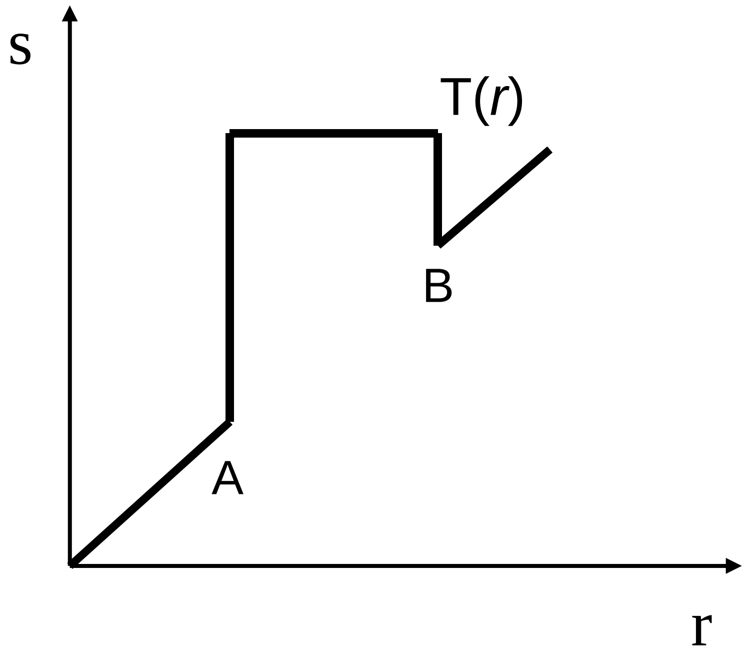
γ =4

γ =5

Gray Level Slicing



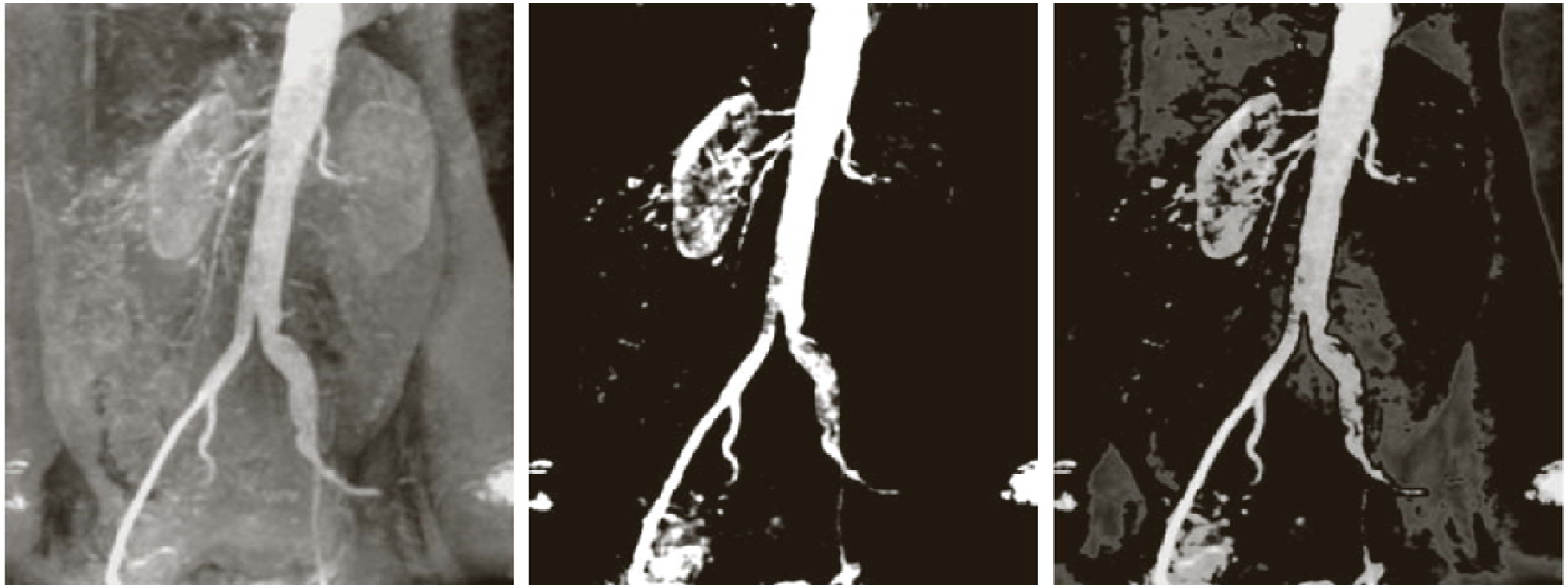
Highlight only the range [A, B]



Preserve other intensities

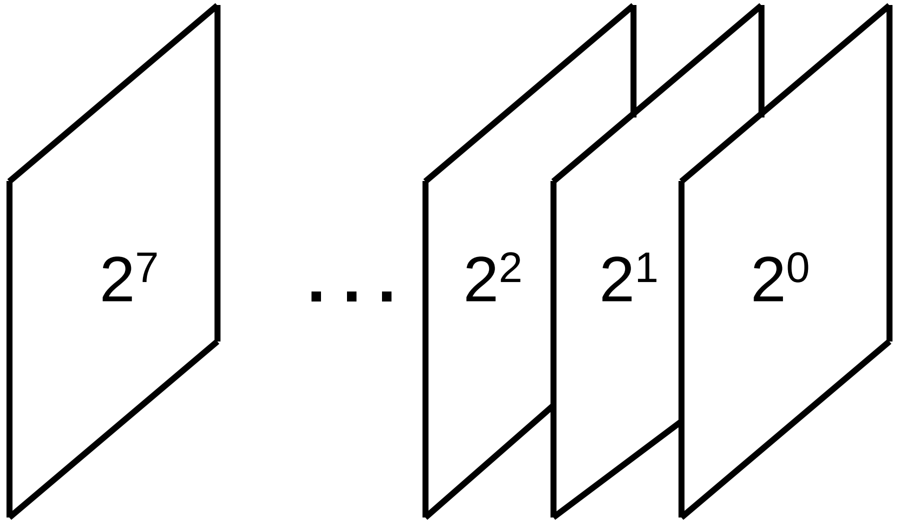
Example: Gray Level Slicing

Case: Aortic angiogram (X-ray)



Original Gray level slicing

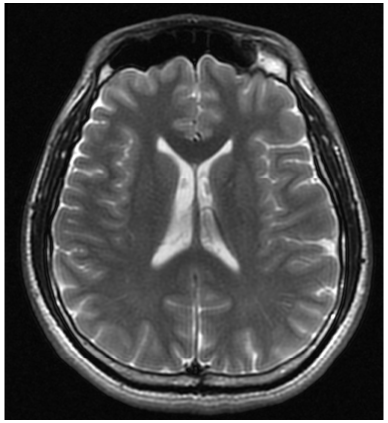
Bit Plane Slicing

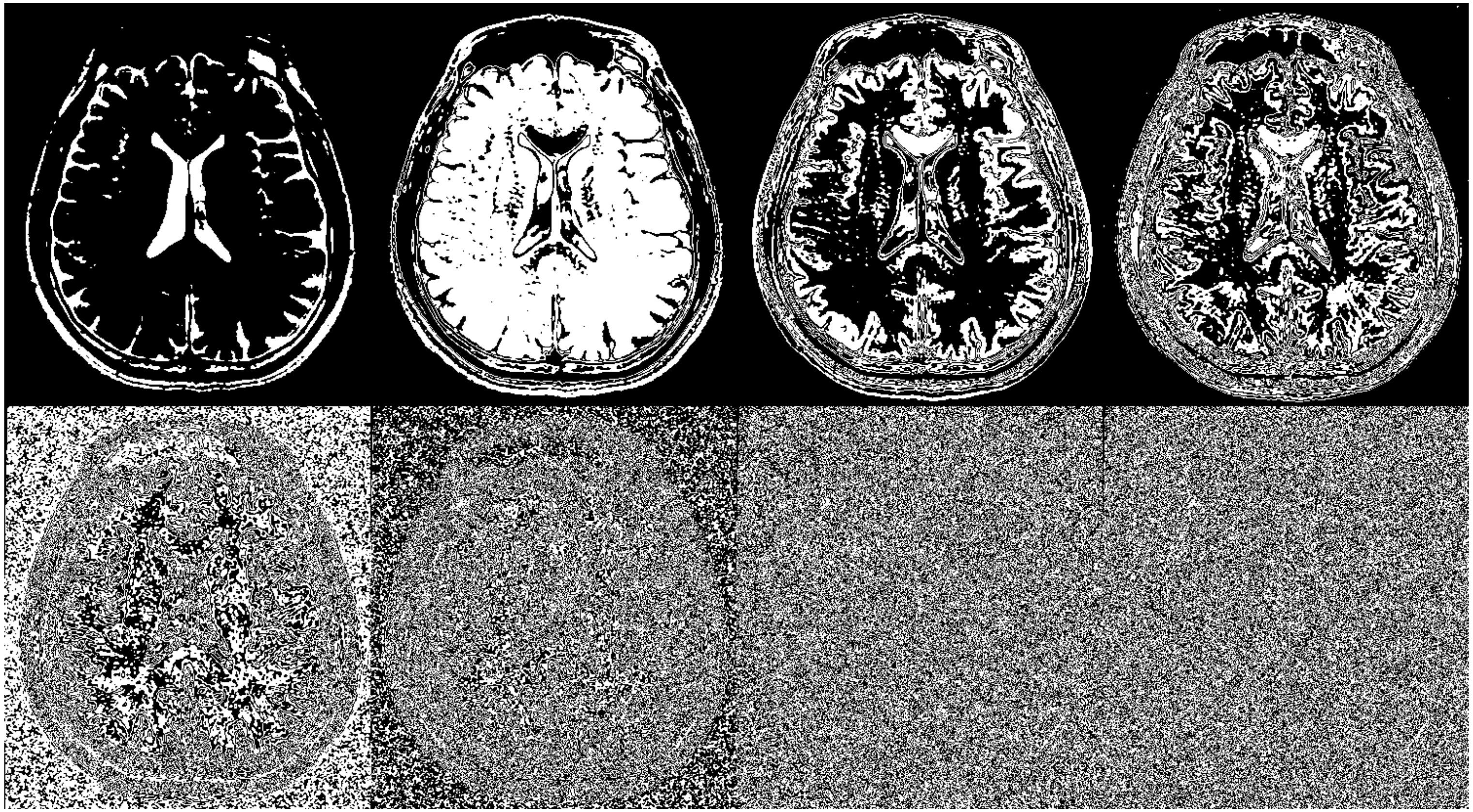


Highlight contributions made by specific bits

Example: Bit Plane Slicing

MSB



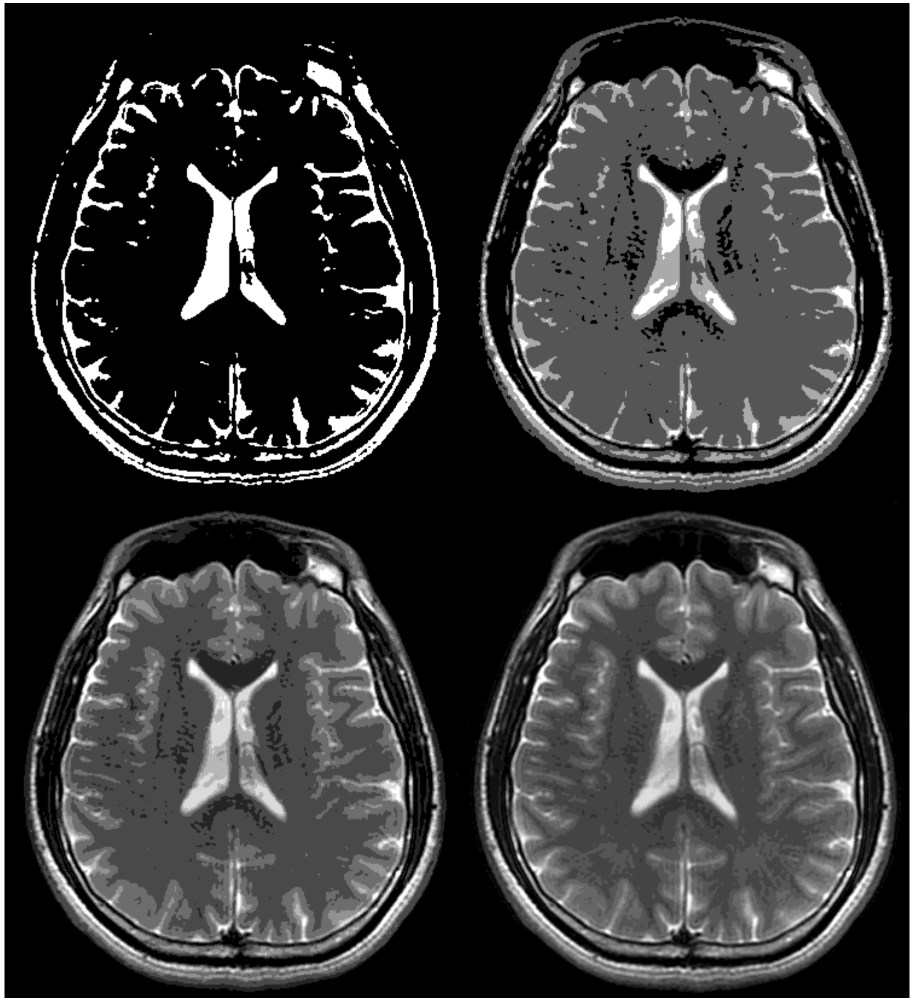


LSB

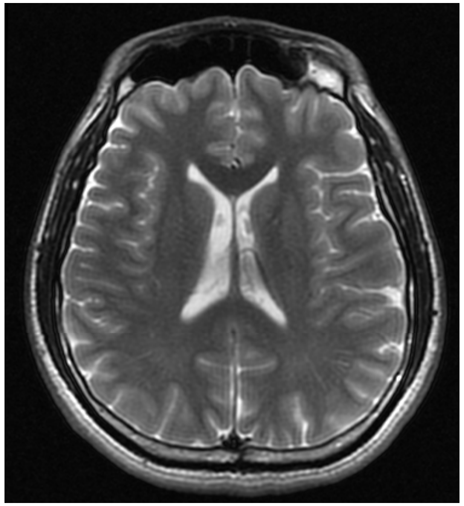
One pixel of gray level 194 → 1 1 0 0 0 0 1 0

延伸一下

2-level 4-level



8-level 16-level



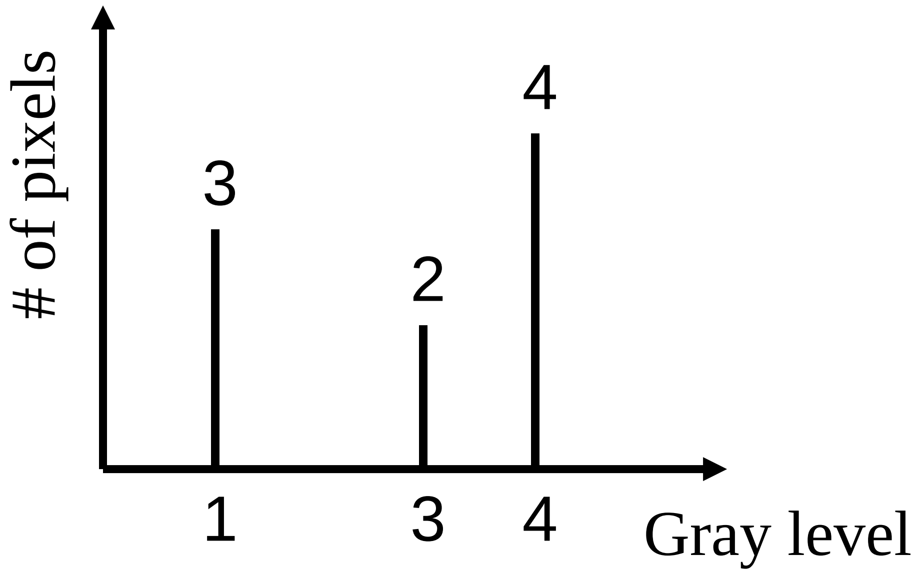
256-level

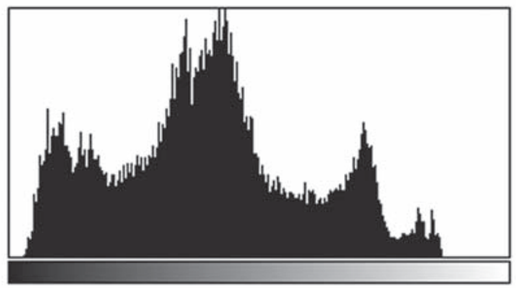
Histogram Processing

- Histogram Equalization
- Histogram Specification / Matching

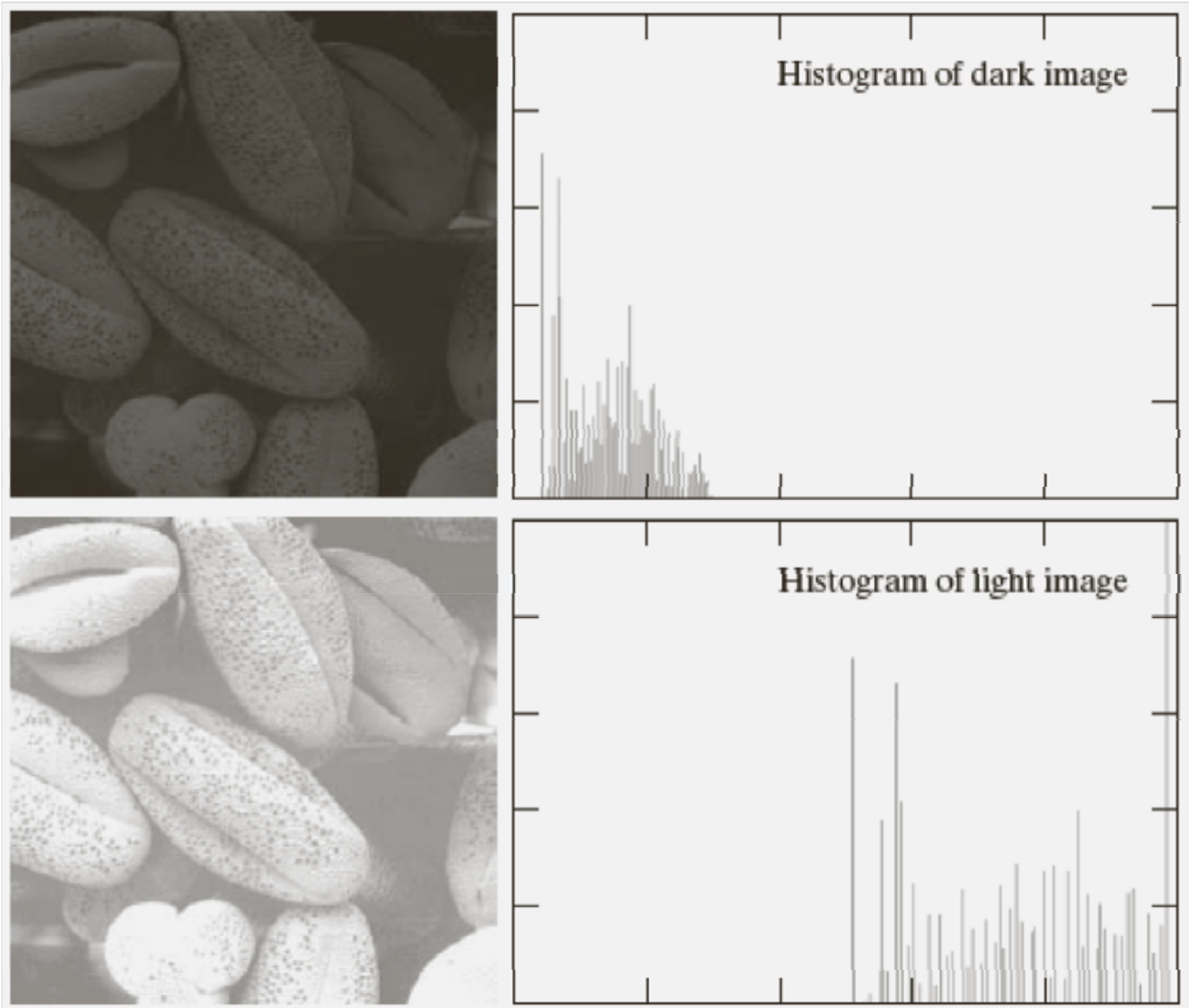
Histogram

1	4	3
4	1	4
1	3	4

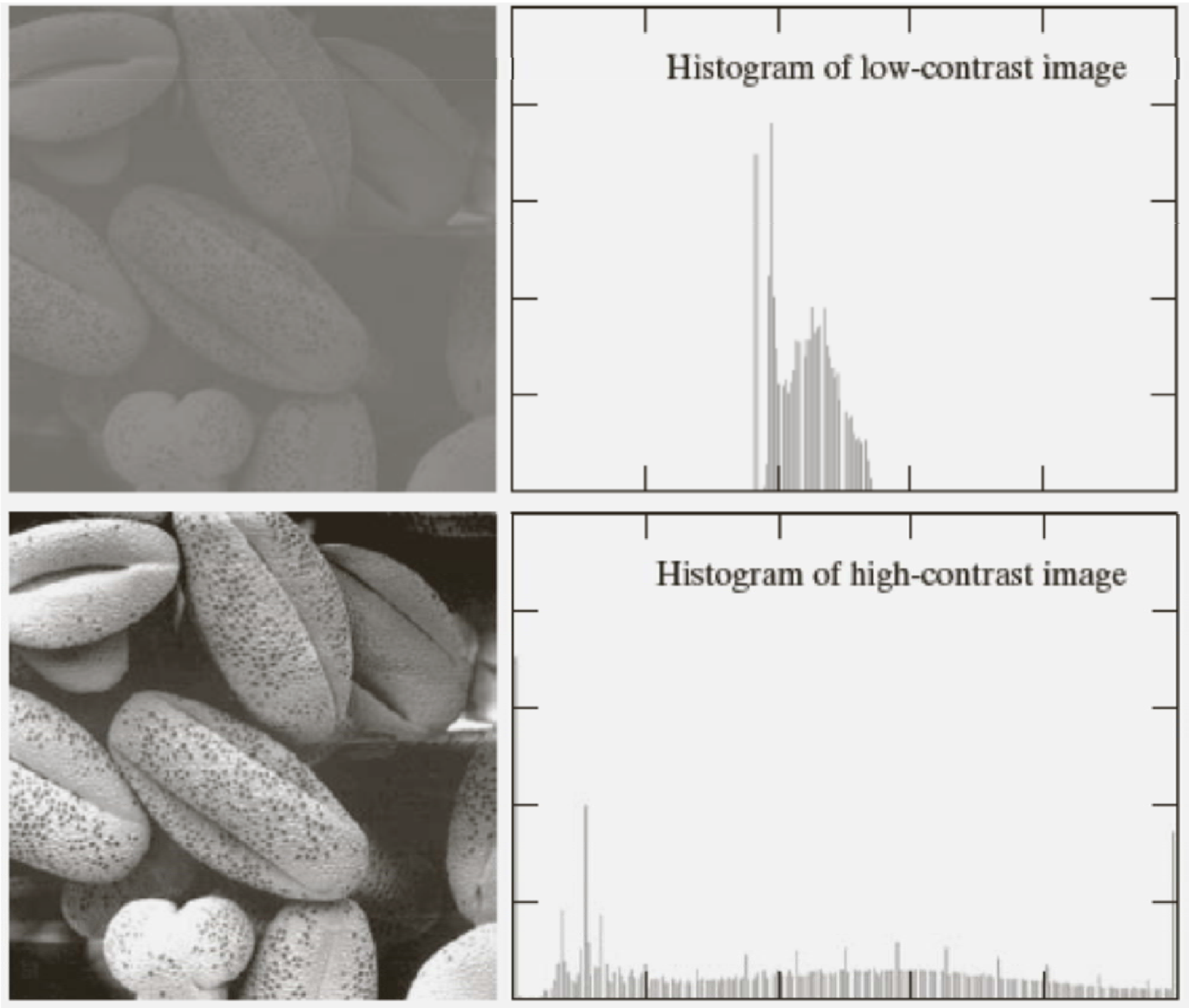




Histograms

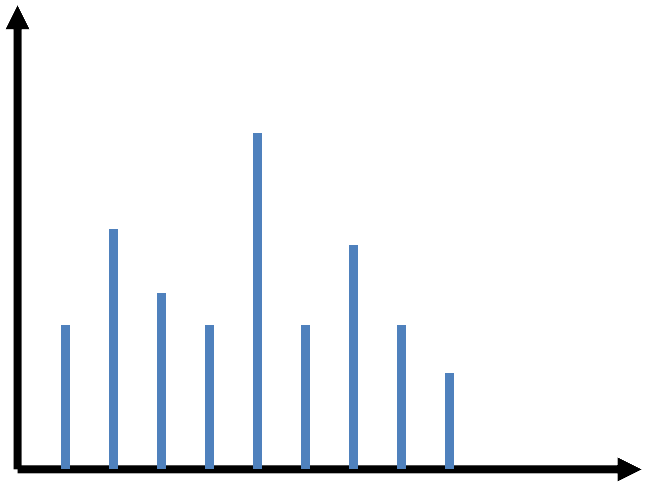


Histograms


Histogram of low-contrast image
Histogram of high-contrast image

Normalized Histogram

$p(r_k) = n_k / n$

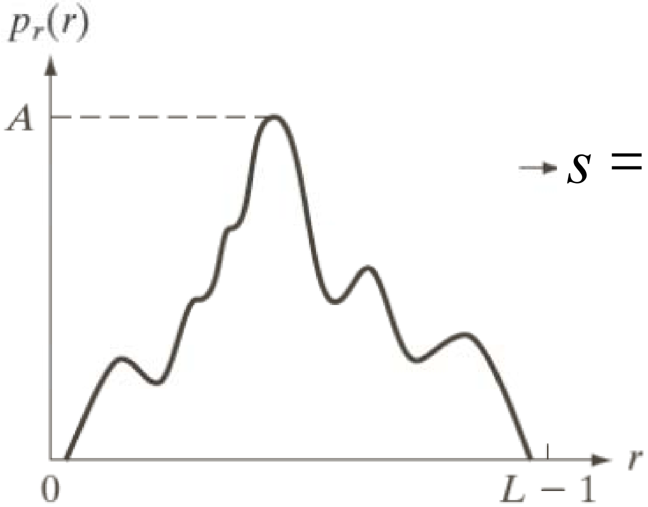


r_k : gray level, integer from 0 to L-1

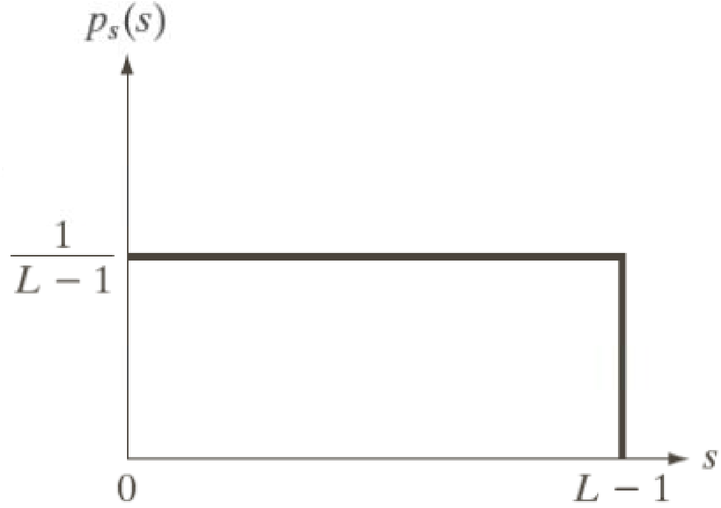
n_k : # of pixels with gray level r_k

n : # of pixels in the image

Histogram Equalization



$\rightarrow s = \mathrm{T}(r) \rightarrow$



We are interested in obtaining a transformation function T( ) which transforms an arbitrary PDF (probability density function) to an <u>**uniform**</u> distribution.

Histogram Equalization

- **Goal**: to obtain $s = T(r)$, where T() is a single valued and monotonically increasing function
  - $r \in [0, L-1]$, input gray level with arbitrary histogram
  - $s \in [0, L-1]$, the transformed gray level with uniform histogram

- $T^{-1}()$, the inverse function of T(), is also single valued and monotonically increasing

Histogram Equalization

- The gray levels in the image can be viewed as random variables taking values in the range [0, L-1]
- Let $p_r(r)$: PDF of input level r
  $p_s(s)$: PDF of s
- Since the number of pixels mapped from r to s is unchanged,

$$p_s(s)ds = p_r(r)dr$$

Histogram Equalization

- Since the desired output histogram (p_s) is uniform,

$$ds = \frac{p_r(r)}{p_s(s)}dr = (L-1)p_r(r)dr$$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Cumulative distribution function (CDF) of *r*

Histogram Equalization

- Consider the discrete functions of histogram, p_r and p_s ,

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{L-1}{N}\sum_{j=0}^{k} n_j, \quad k = 0,1,\ldots,L-1$$

n_k : # of pixels with gray level r_k

N : # of pixels in the image

Example: Equalization

r	0	1	2	3	4	5	6	7
n_k	1	7	21	35	35	21	7	1

$N = 128$

Goal ↓

s	0	1	2	3	4	5	6	7
n_k	16	16	16	16	16	16	16	16

Example: Equalization

$$s_k = \frac{L-1}{N}\sum_{j=0}^{k} n_j, \quad k = 0,1,\ldots,L-1$$

r	0	1	2	3	4	5	6	7
n_k	1	7	21	35	35	21	7	1

s = (8-1)/128 x [1 8 29 64 99 120 127 128]
= [0 0 2 4 5 7 7 7] = T(r)

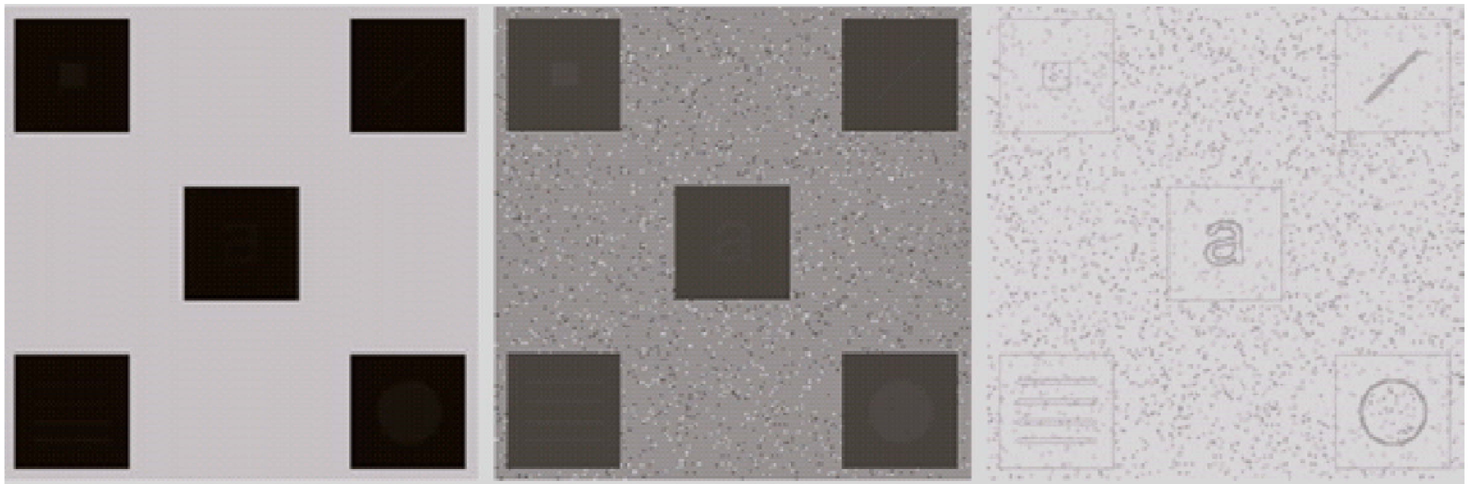
s	0	1	2	3	4	5	6	7
n_k	8	0	21	0	35	35	0	29

Equalization

- Histogram equalization is very useful for low contrast images.
- Intuitively, an image, whose pixels tend to occupy the entire range of intensity levels and to distribute uniformly, will have an appearance of high contrast.

Histogram matching / specification

- Uniform histogram is not always the best !
- For example, images with detail might hidden in dark region.
- To design the specific histogram



Matching / Specification

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

p_r : original histogram

p_z(z) : desired histogram

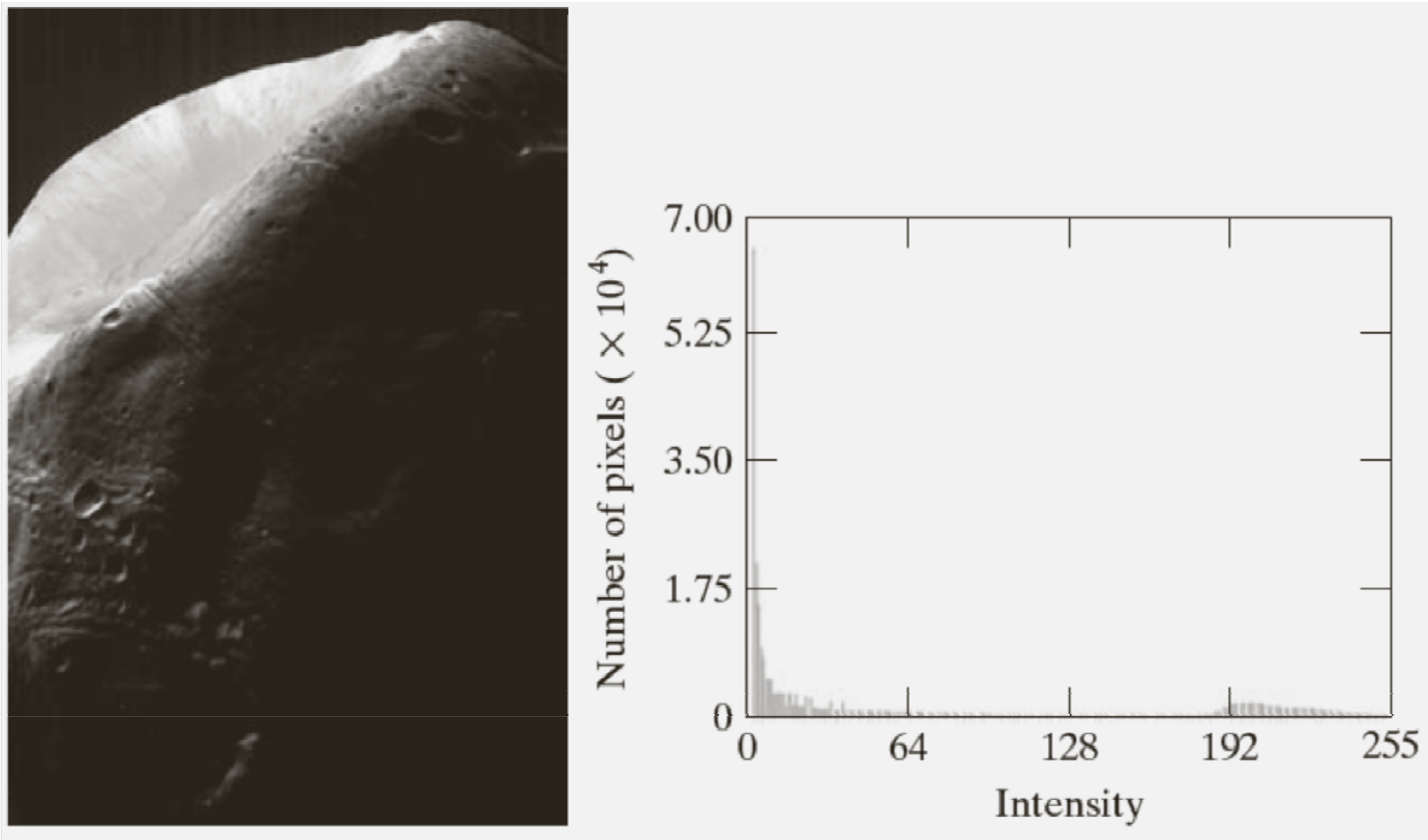
Steps:

(1) Equalize the levels of original image $s = T(r)$

(2) Specify the desired $p_z(z)$ and obtain $s = G(z)$

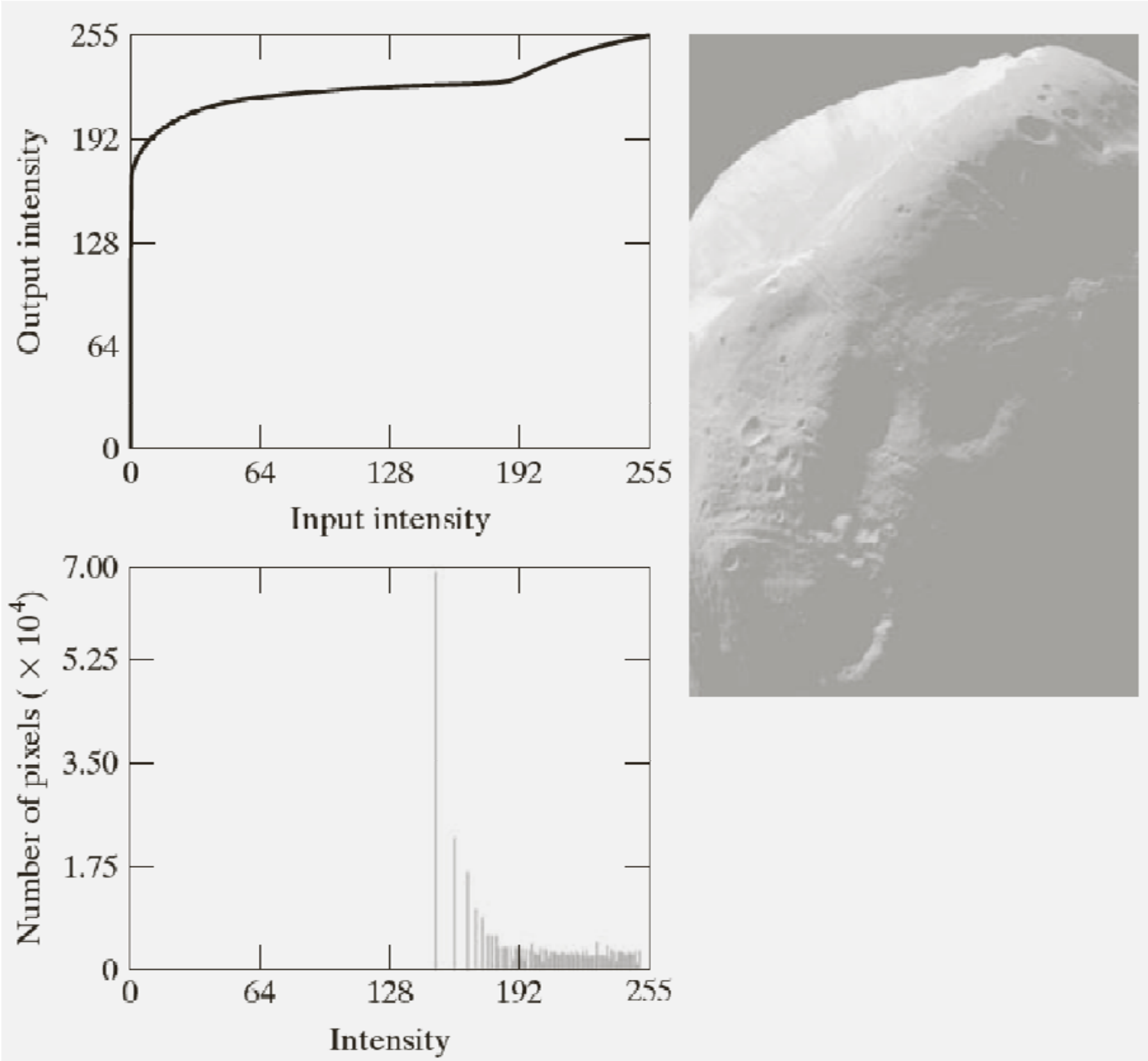
(3) Apply $z = G^{-1}(s)$ to the levels s obtained in step 1

Example: histogram specification



Original image and histogram

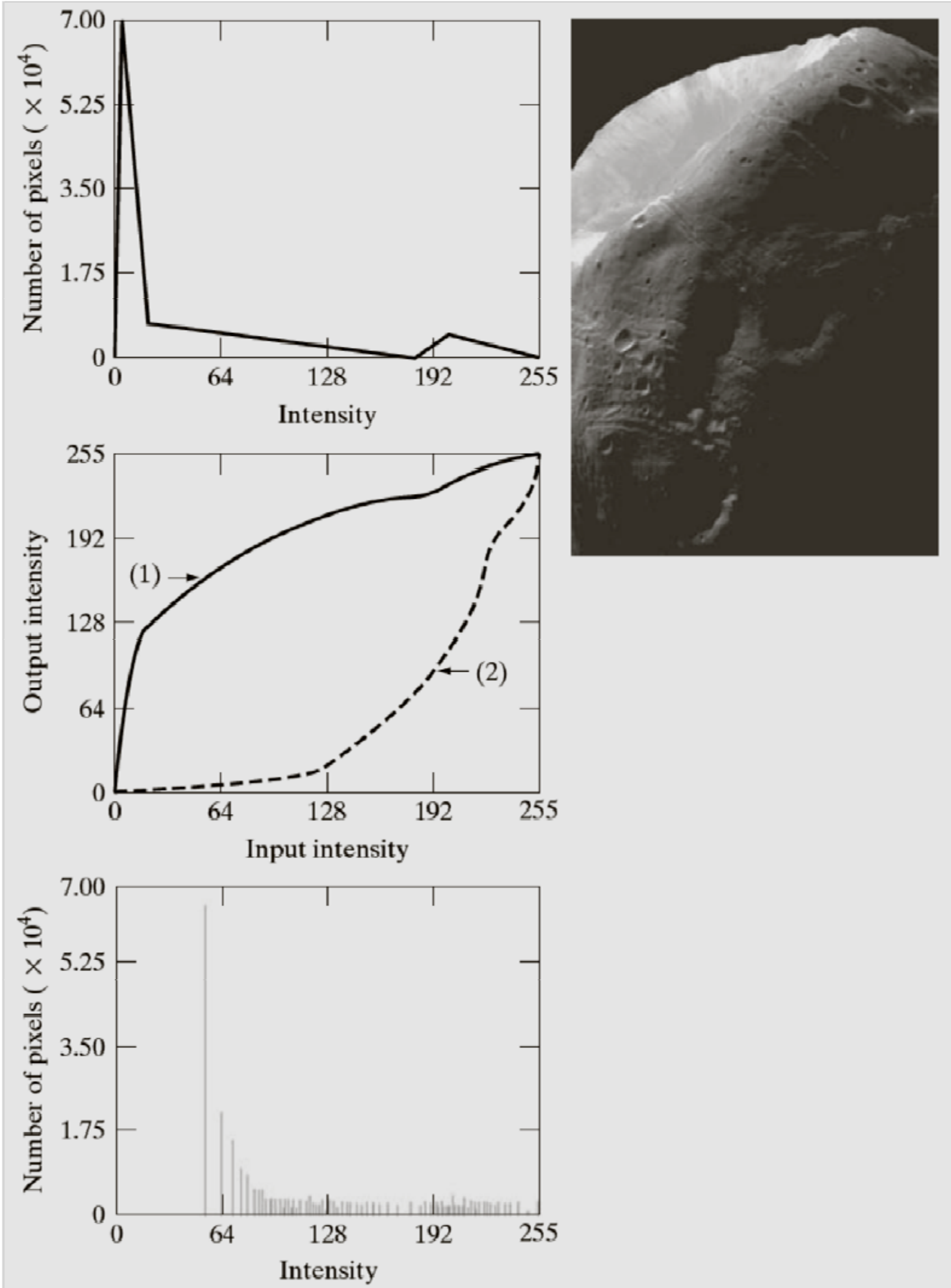
After automatic equalization



Specified histogram

(1) $G(z)$ and
(2) $G^{-1}(s)$

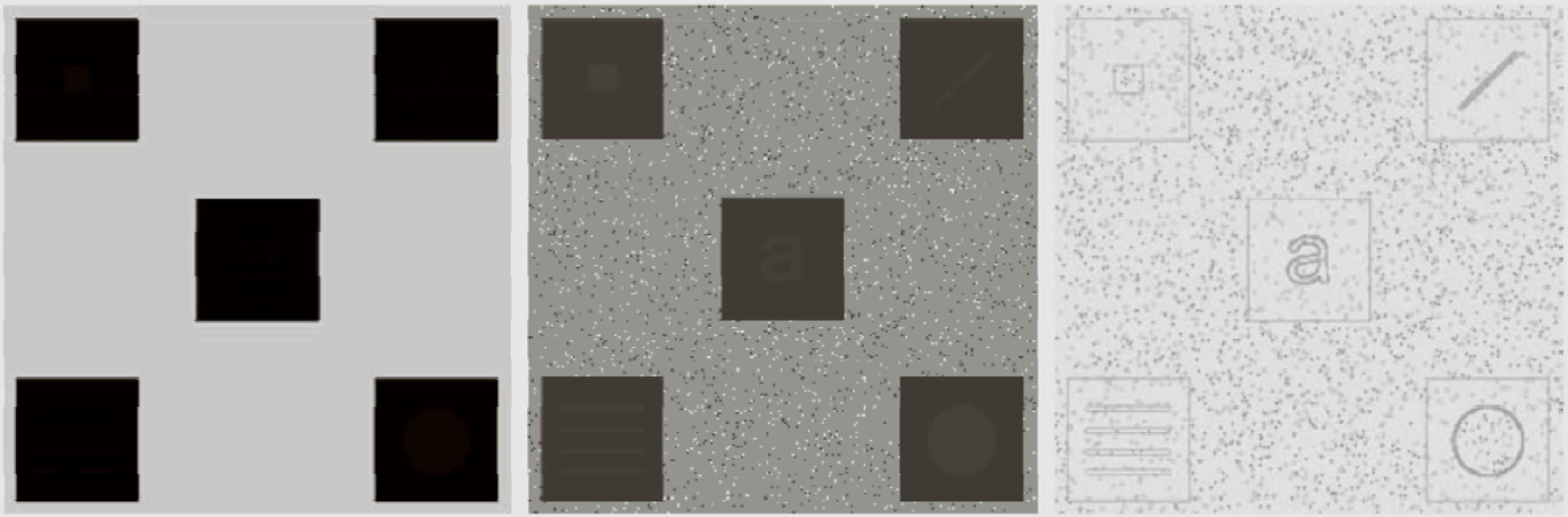
Actual histogram



Local equalization

- The previously discussed two methods are global.
- Can equalization be local?
- Define a neighborhood to repeat the procedure of uniform or specified equalization.

Local histogram equalization



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3.

Local contrast enhancement

$$g(x, y) = A(x, y)[f(x, y) - m(x, y)] + m(x, y)$$

where $m(x, y)$: local mean

$A(x, y) = kM/\sigma(x, y)$, $0 < k < 1$

$\sigma(x, y)$: local standard deviation

M: global mean

Areas with low contrast

→ Larger gain $A(x,y)$

Example: Local contrast enhancement

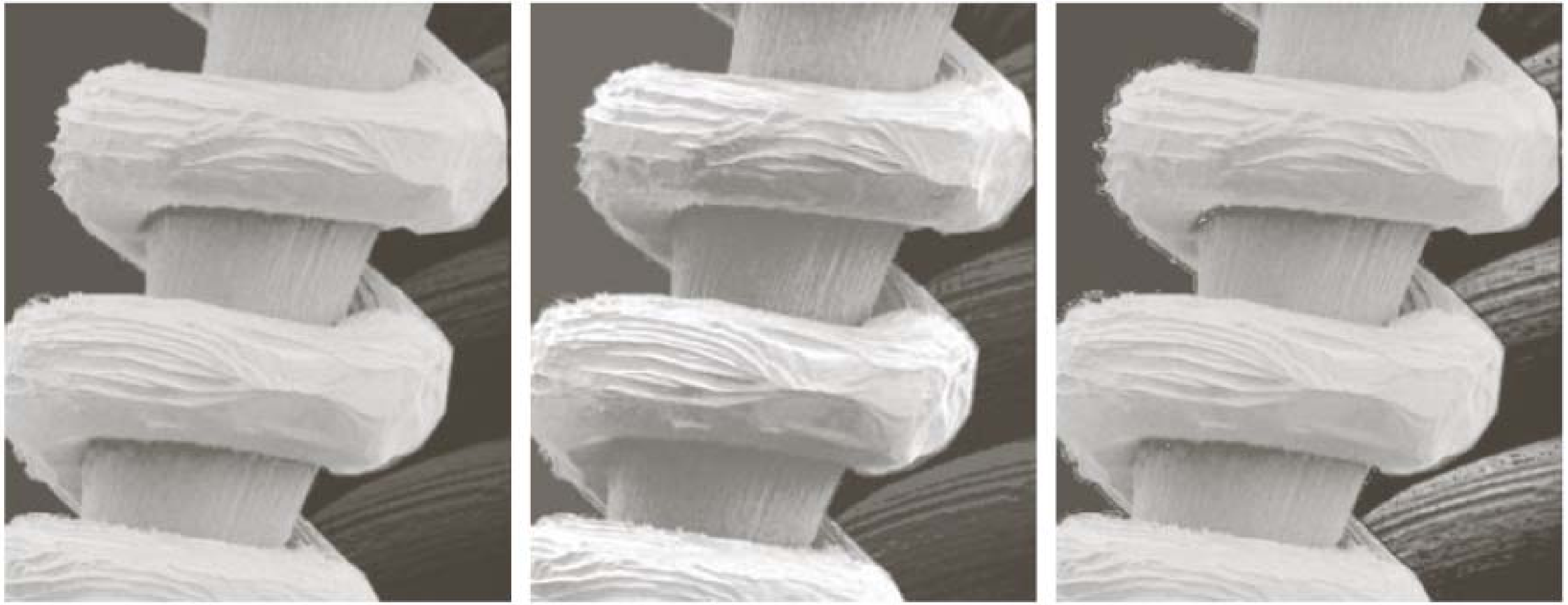


FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Other useful operations

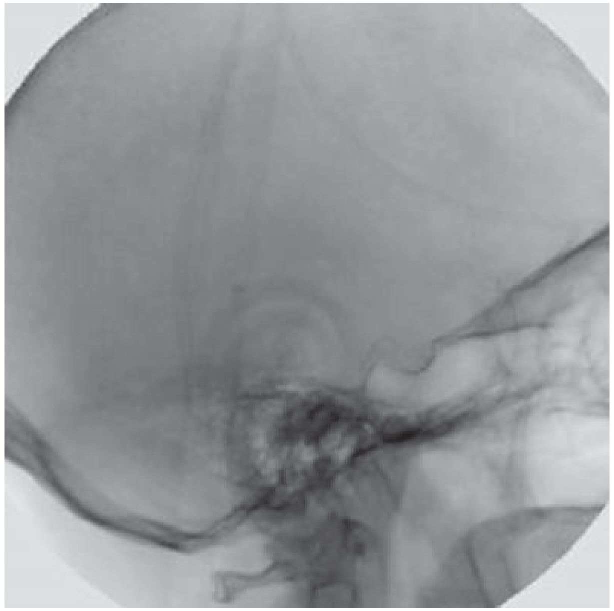
- Simple and commonly used in routine!!
- Algebraic operation
  - Image Subtraction
  - Averaging
- Logical operation
  - Selection of region of interest (ROI)
  - AND / OR

Image subtraction

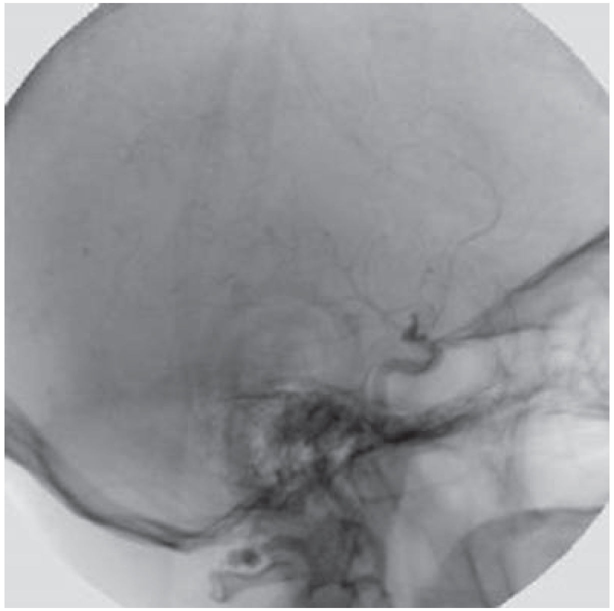
- Application in medical imaging –"mask mode radiography"
- $h(x, y)$ is the mask, e.g., an X-ray image of part of a body
  $f(x, y)$: incoming image after injecting a contrast medium

$$g(x, y) = f(x, y) - h(x, y)$$

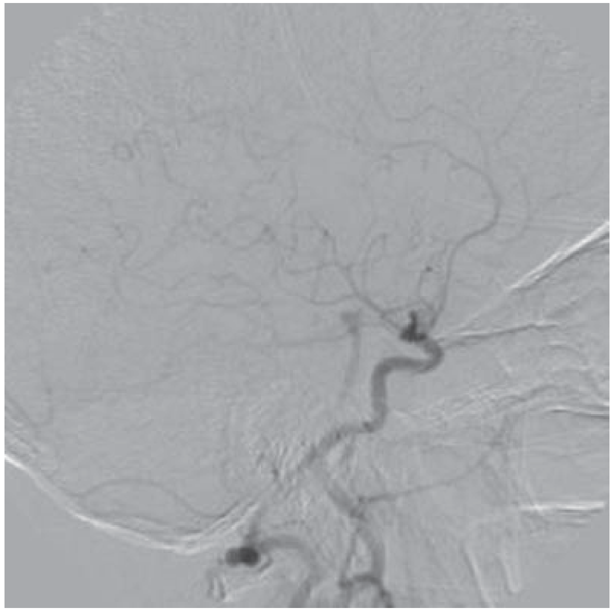
Image subtraction



Pre-contrast



Post-contrast



Subtraction Angiography

Image Average

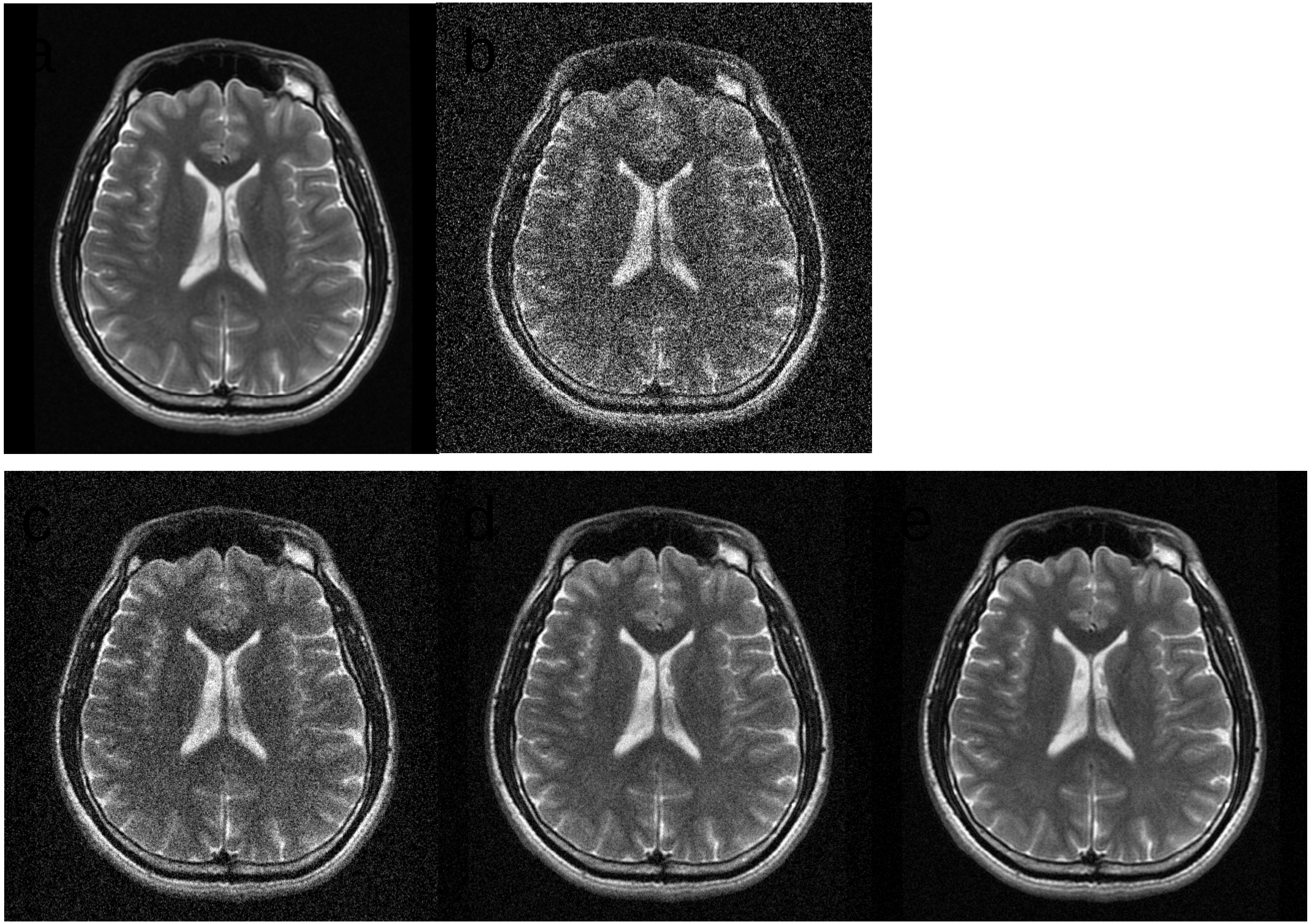
$$g(x, y) = f(x, y) + \eta(x, y)$$

$$\overline{g}(x, y) = \frac{1}{K}\sum_{i=1}^{k} g_i(x, y)$$

$$E\{\overline{g}(x, y)\} = f(x, y)$$

$\eta\ (x, y)$: random noise zero mean

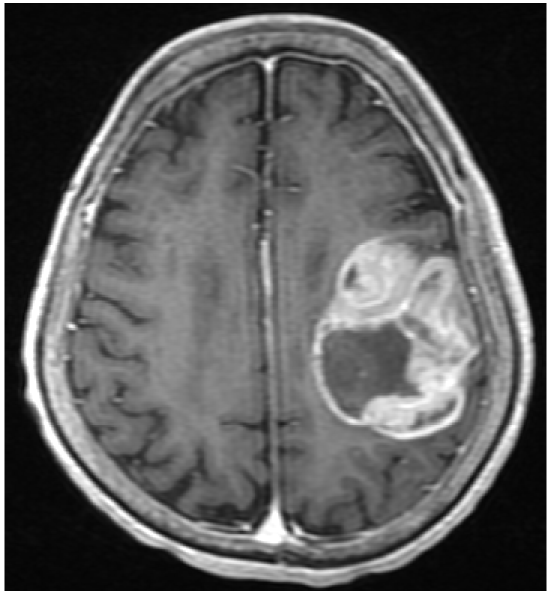
SNR ??



(a) An MRI of axial brain is shown in 256 gray level. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c-e) Results of averaging number = 4, 16, and 64.

Masking

Case: MRI of brain with
glioblastoma multiforme



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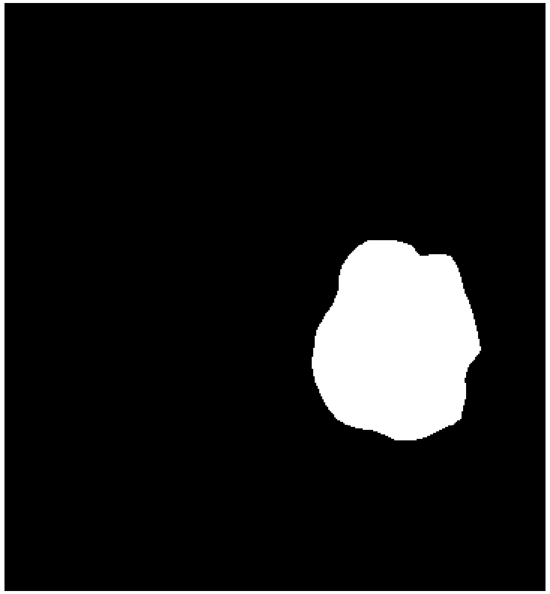
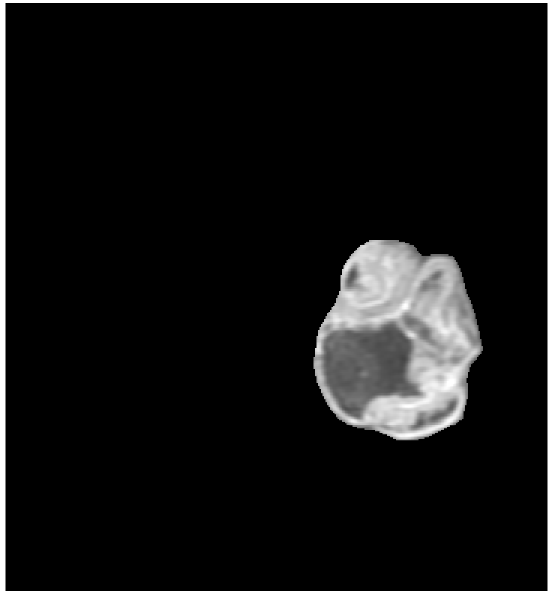


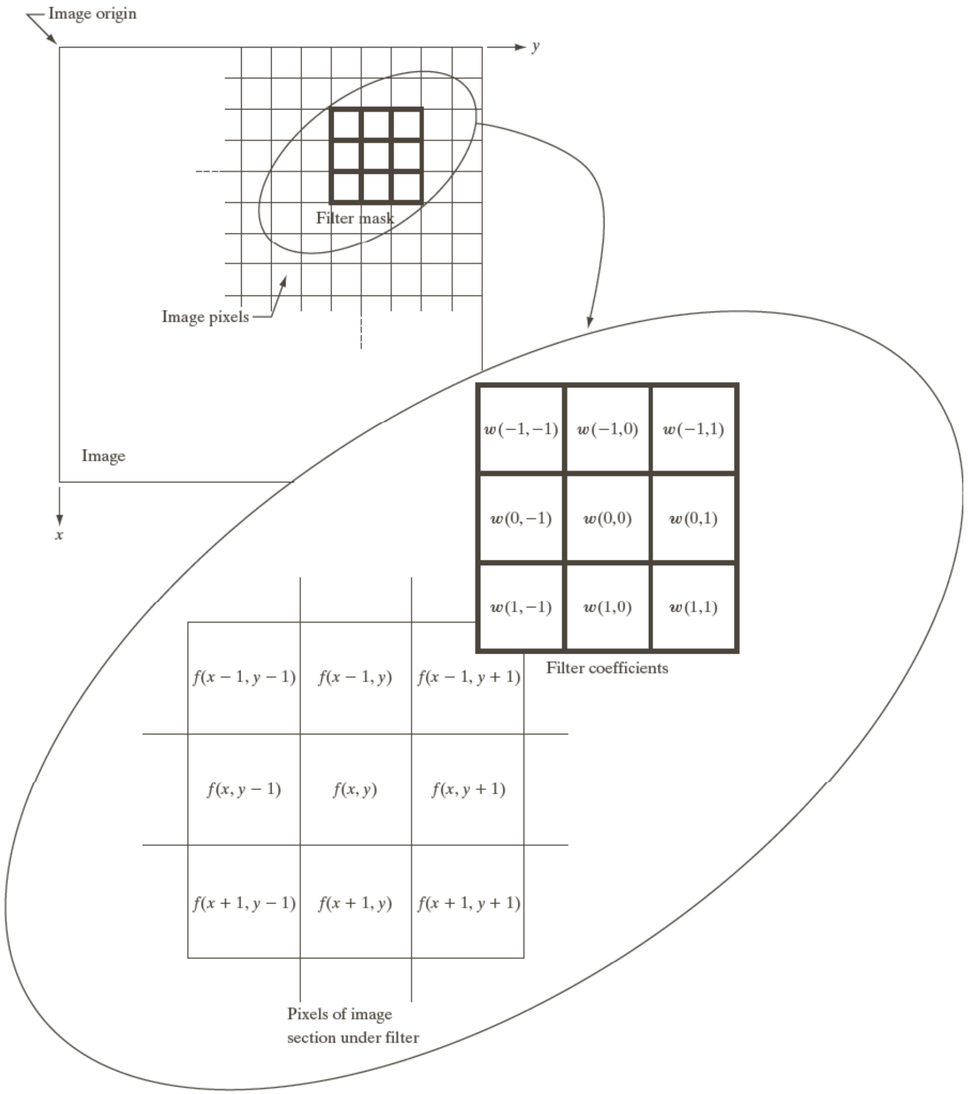
=



Mask of ROI

Filtering in spatial domain

Spatial filtering

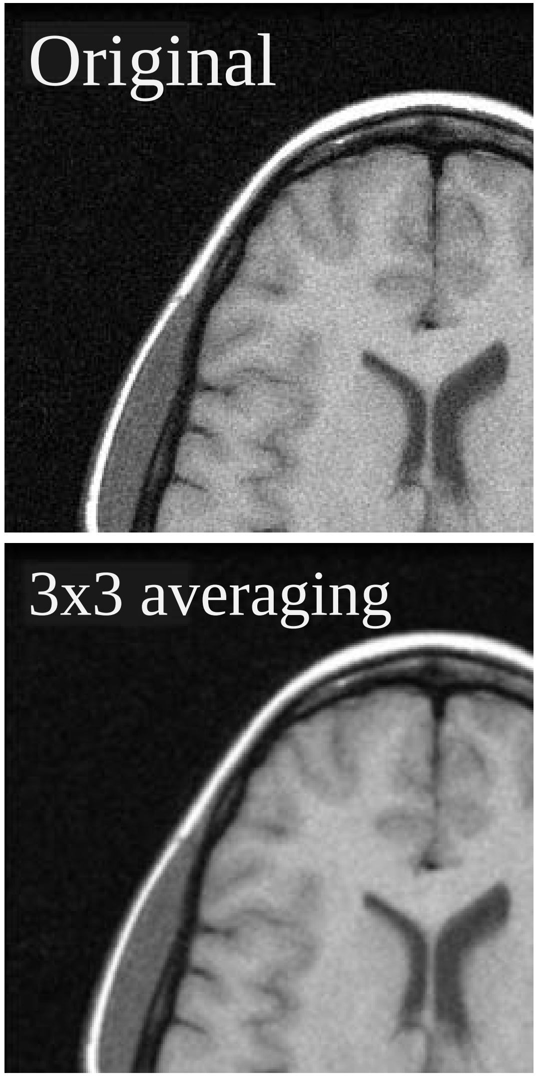


Replace $f(x, y)$ with

$$g(x,y) = \sum_i w_i f_i$$

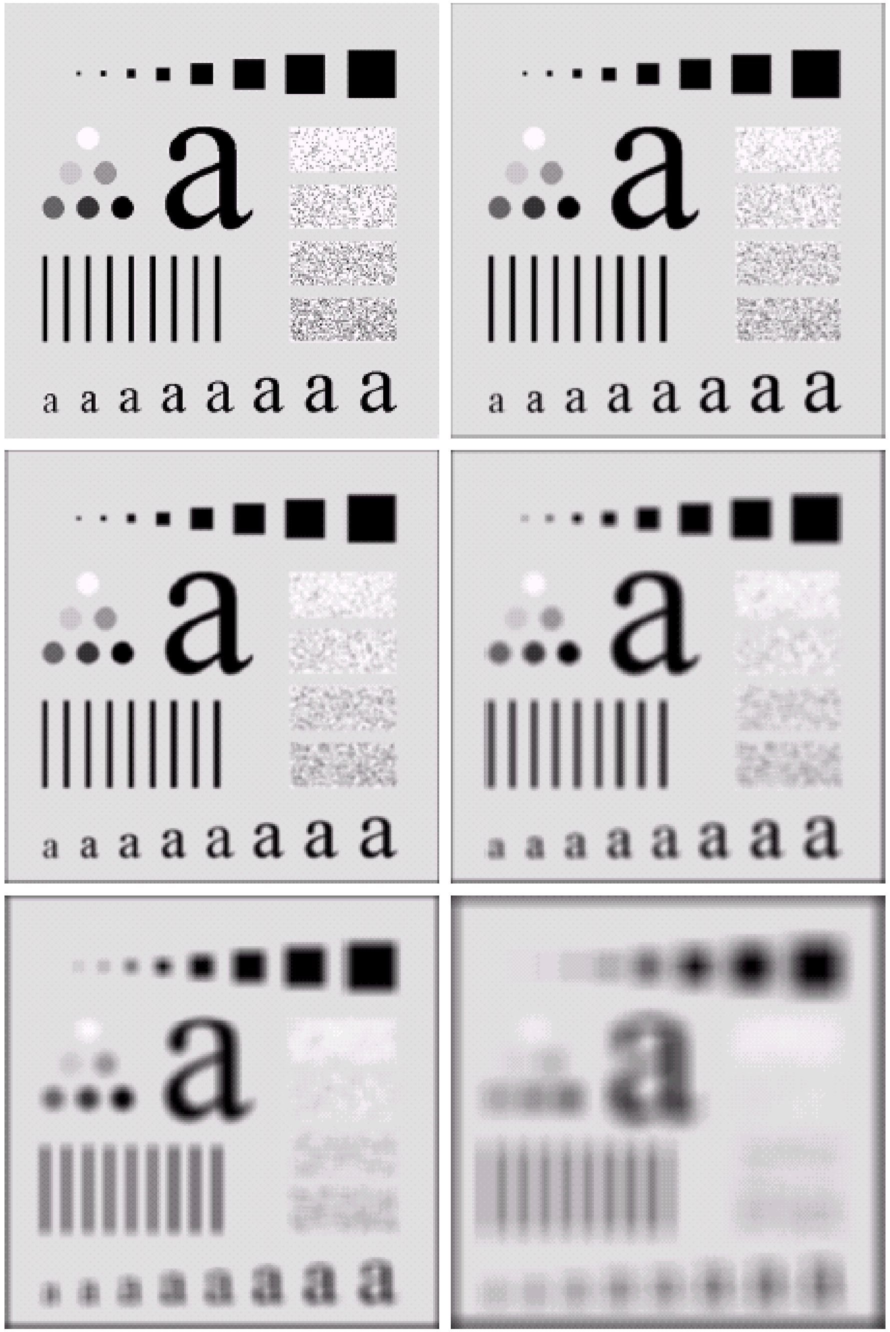
Low pass filtering

- Low-pass filter (LPF)
  - Reduce additive noise
  - Blur/smooth the image
  - Sharp details lost
  - Ex: local averaging



Original (500 × 500 pixels)

3×3



6×6

9×9

15×15

35×35

Median Filter

- Replace $f(x, y)$ with median$[f(x', y')]$
- Example:

10	20	20
20	15	20
25	20	100

→ Median (10, 15, 20, 20, 20, 20, 20, 25, 100) = 20

→ Replace "15" with "20"

Median Filter

- Non-linear filter
- Useful in eliminating intensity spikes. (salt-and-pepper noise)
- Better at preserving edges.

Median Filter

Original and with salt & pepper noise

% imnoise(image, “salt & pepper”);

Median Filter

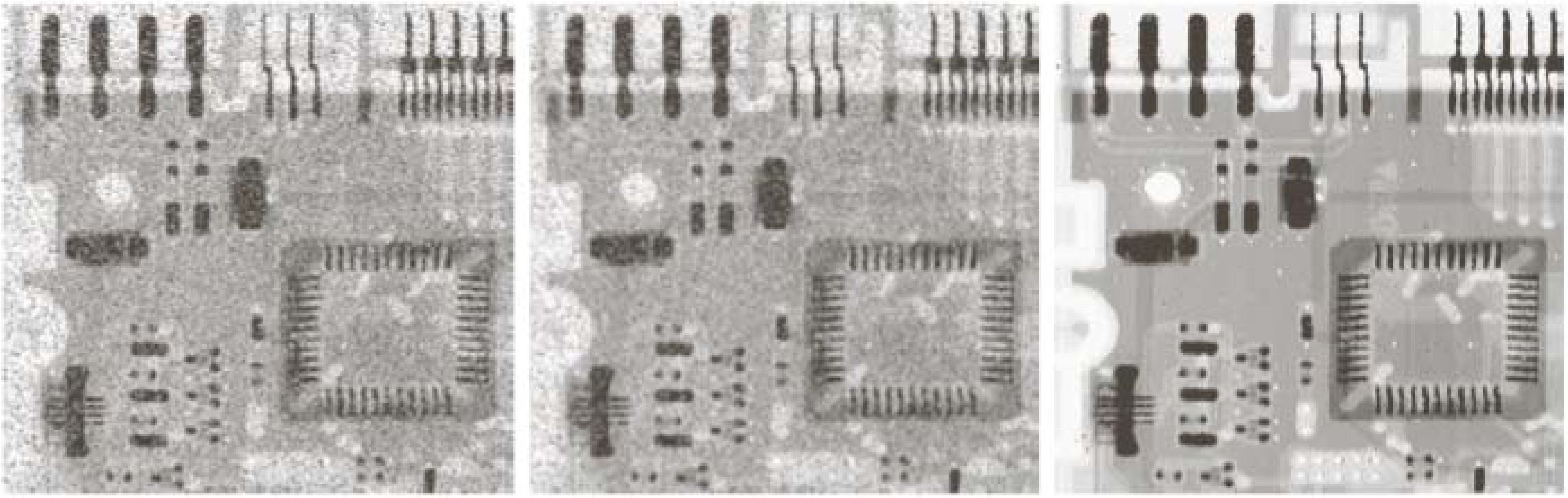
Original

Noise added

Local Averaging

Median filtered

Median Filter



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

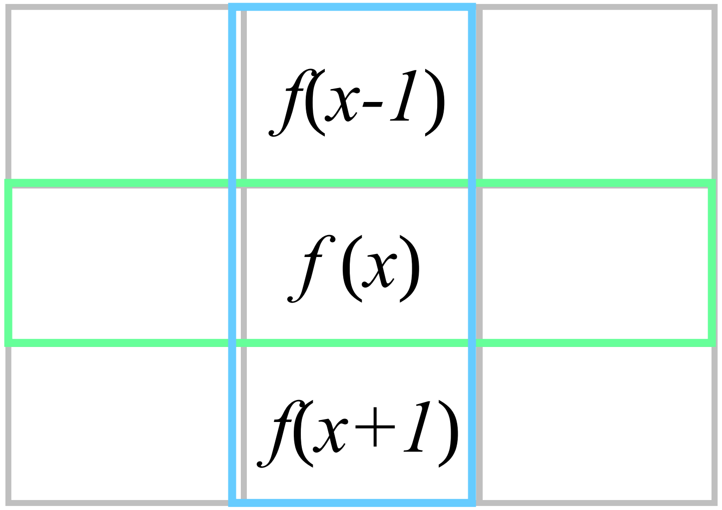
High pass filter

- Enhance finer image details, such as edges.
- Detect region or object boundaries.
- Smoothing (LPF) v.s. sharpening (HPF)

1-D derivative

- First derivative

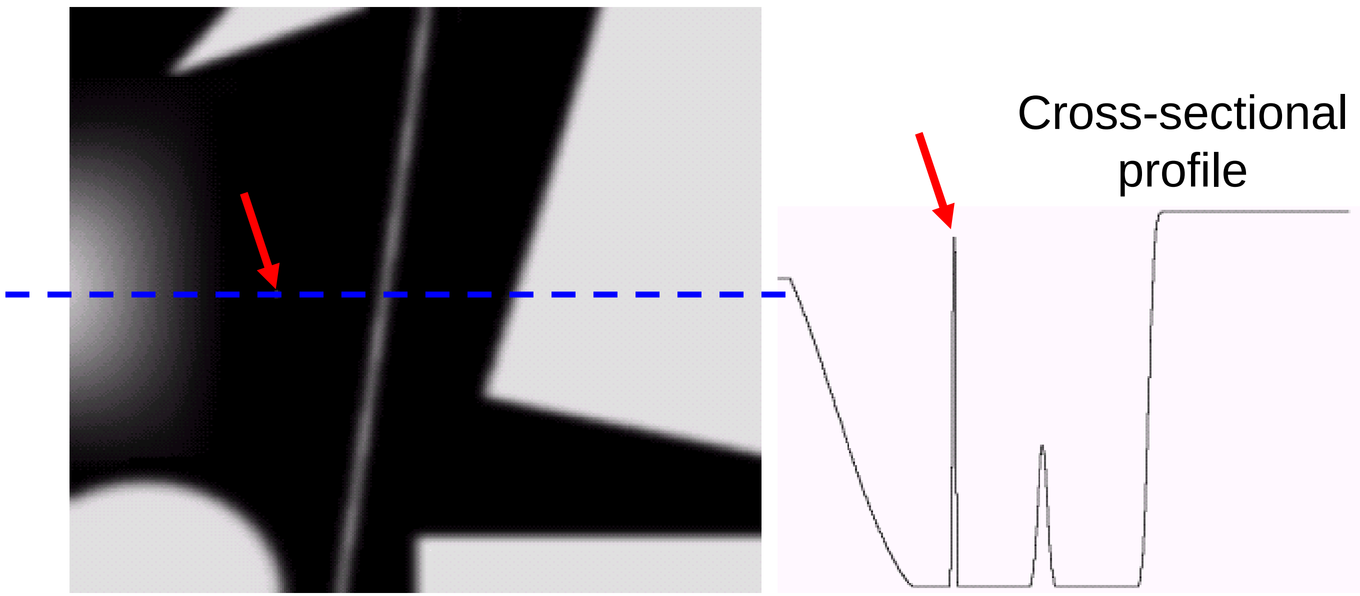
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$



- Second derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Example: 1-D derivative



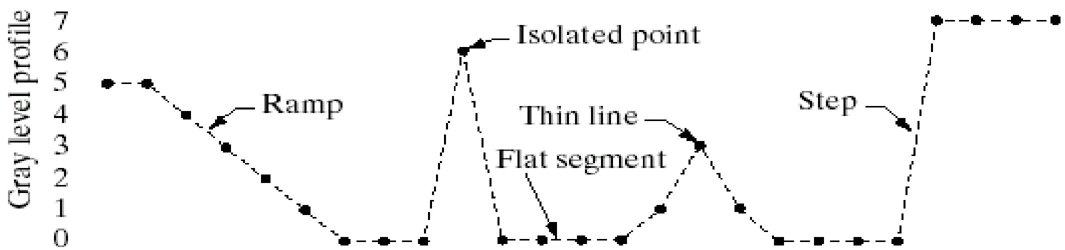


Image strip	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	•	•
First Derivative		−1	−1	−1	−1	−1	0	0	6	−6	0	0	0	1	2	−2	−1	0	0	0	7	0	0	0			
Second Derivative		−1	0	0	0	0	1	0	6	−12	6	0	0	1	1	−4	1	1	0	0	7	−7	0	0			

Laplacian: second derivative

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

0	0	0
1	-2	1
0	0	0

0	1	0
0	-2	0
0	1	0

0	1	0
1	-4	1
0	1	0

More Laplacian operators…

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

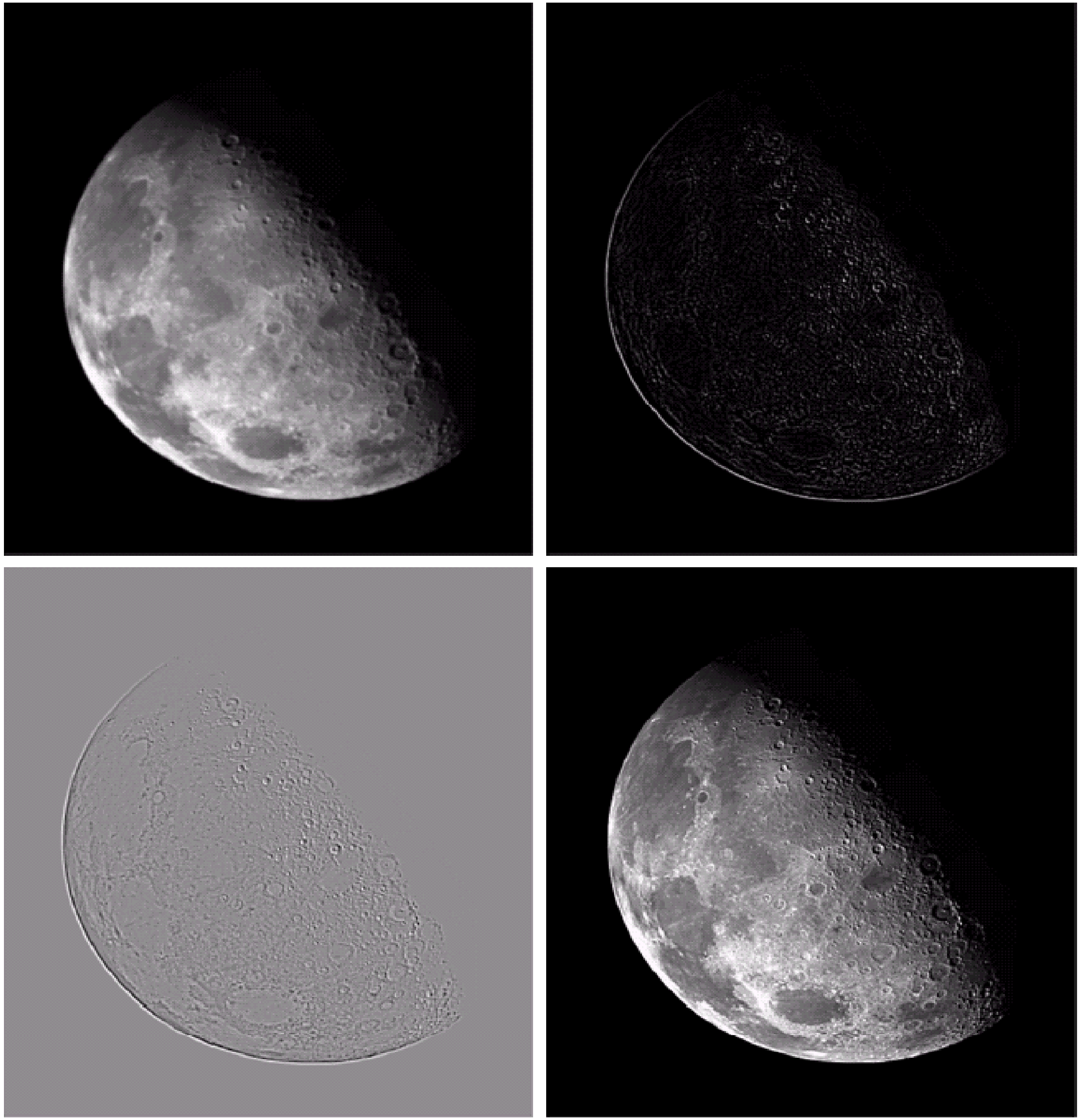
Laplacian based edge detectors

- Rotationally symmetric, linear operator
- Second derivatives => sensitive to noise
- Increase the contrast at the locations of gray-level discontinuities.

Sharpening with the Laplacian

- Laplacian filters can be used to detect edges.
- It can also be used to sharpen the image,

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{If the center coefficient of the Laplacian mask} < 0 \\ f(x,y) + \nabla^2 f(x,y) & \text{If the center coefficient of the Laplacian mask} > 0 \end{cases}$$



Unsharp masking

- Subtract Low pass filtered version (f_{LPF}) from the original (f)

$$f_s(x, y) = f(x, y) - f_{LPF}(x, y)$$

- Emphasizes high frequency information – unsharp masking

$$g(x, y) = f(x, y) + f_s(x, y)$$

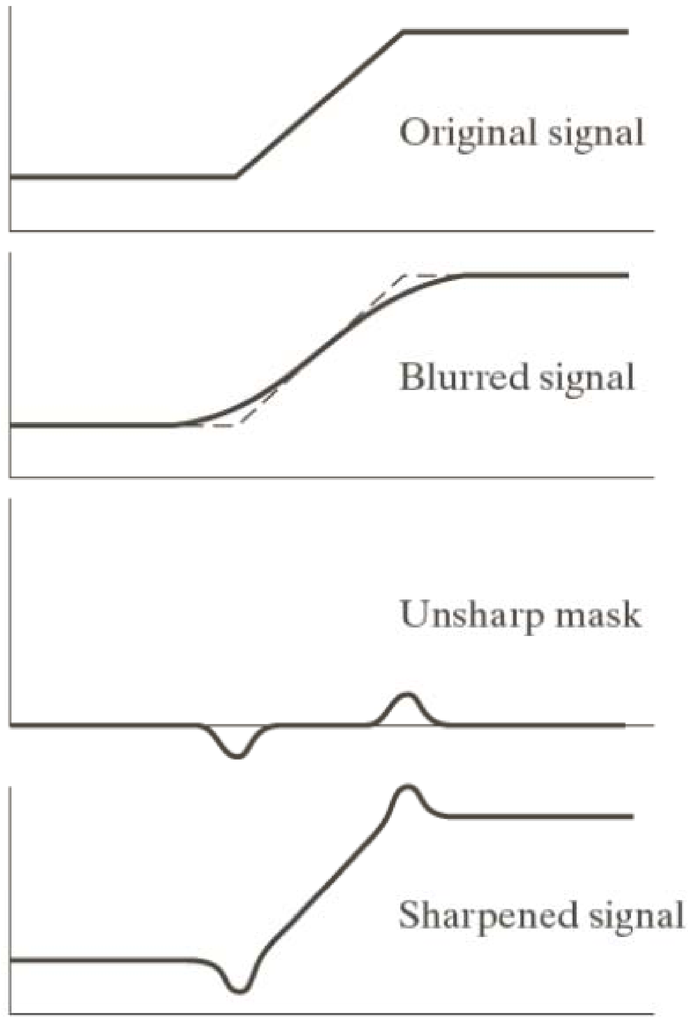
High-boost filtering

- Highboost filter (k >1)

$$g(x,y) = f(x,y) + k \cdot f_s(x,y)$$

- Compare with the generalized sharpening form using Laplacian

$$g(x,y) = \begin{cases} Af(x,y) - \nabla^2 f(x,y) & \text{If the center coefficient of the Laplacian mask} < 0 \\ Af(x,y) + \nabla^2 f(x,y) & \text{If the center coefficient of the Laplacian mask} > 0 \end{cases}$$



Original

Blurring with a Gaussian filter

Unsharp mask

Unsharpened image ($k = 1$)

Highboost filtering ($k = 4.5$)

Sobel gradients

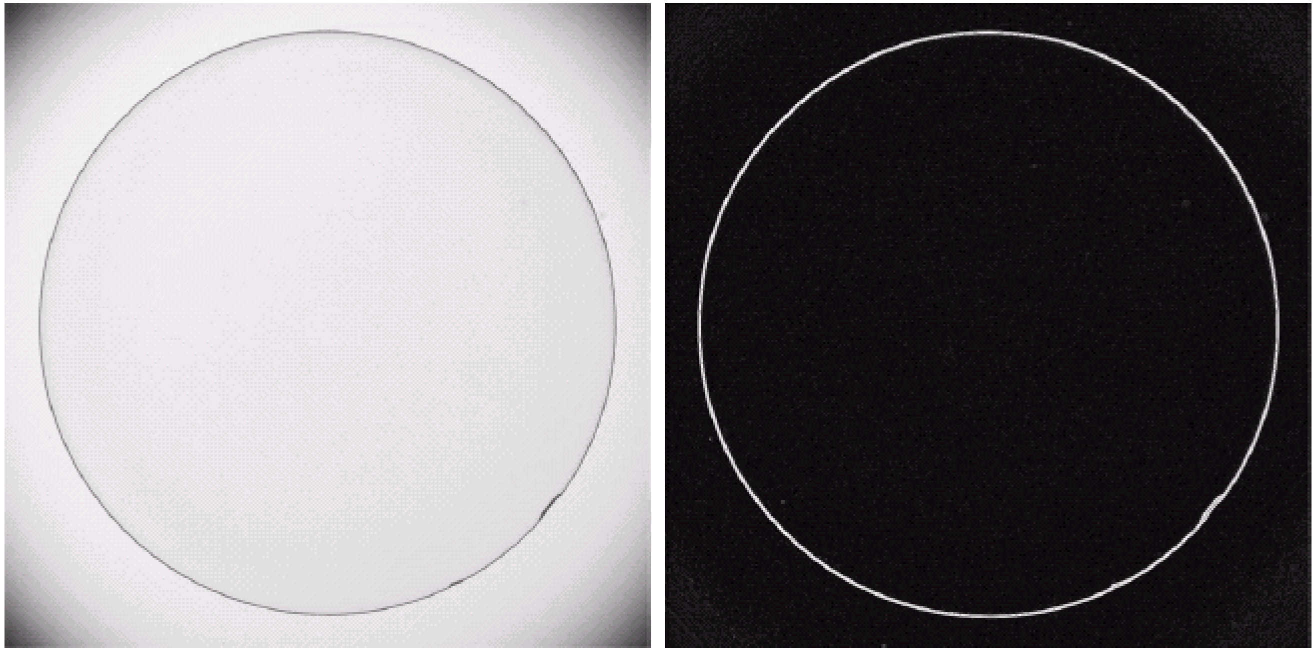
- The Sobel gradients, g_x and g_y

-1	-2	-1
0	0	0
1	2	1

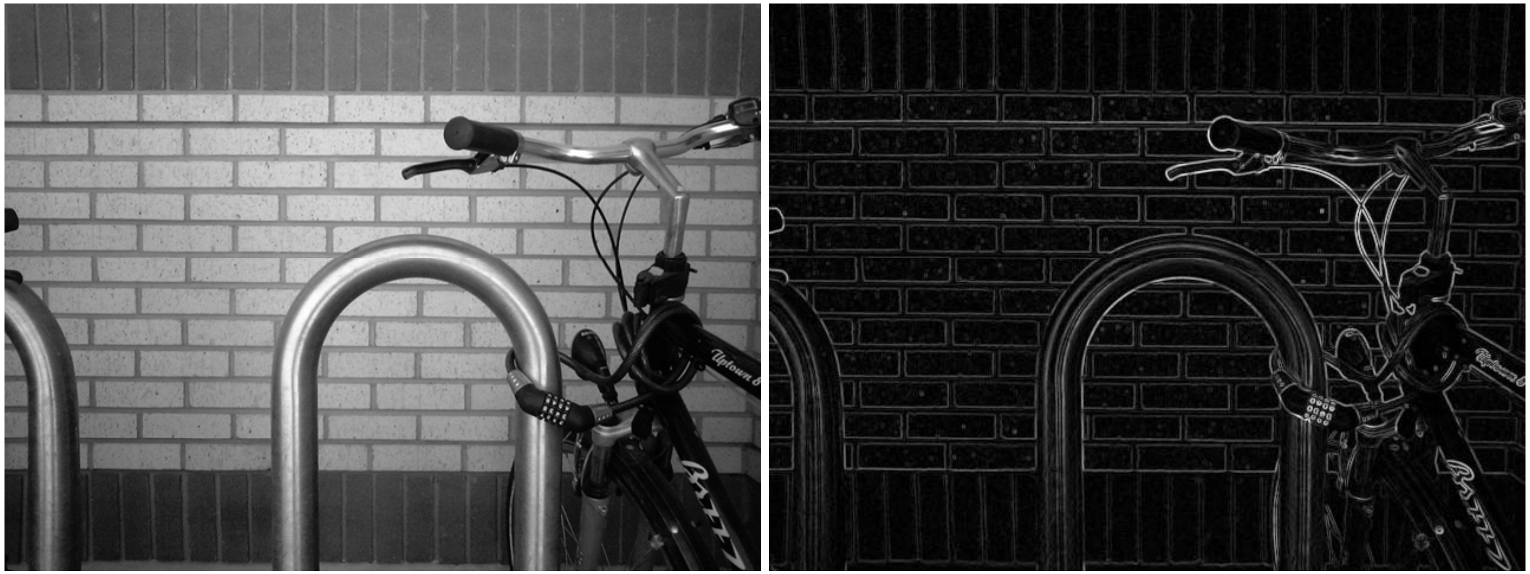
-1	0	1
-2	0	2
-1	0	1

- Magnitude of gradients

$$\|\nabla f\| = \sqrt{g_x^2 + g_y^2}$$



Left: Grayscale image Right: Sobel gradient



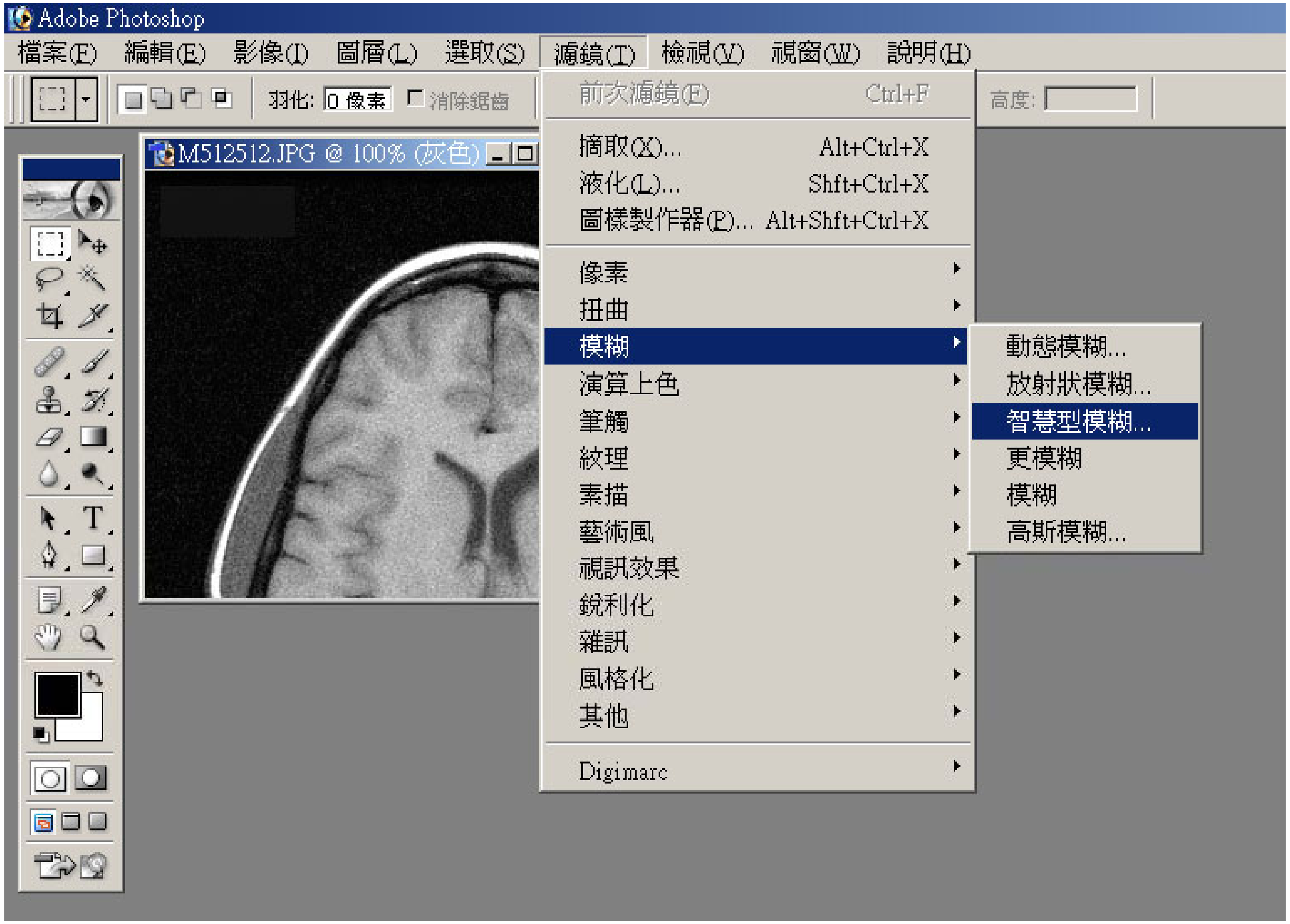
Gradient Process

- Edge detection
- Constant or slowly varying shades are eliminated
- Automated inspection
- Related topic – segmentation, registration

稍微打個岔 ...

- Adobe Photoshop (繪圖藝術軟體)
- 濾鏡功能選項
  - 模糊、更模糊、高斯模糊 ...
  - 銳利化、更銳利化...
  - 雜訊中和、增加雜訊...
- 幾乎都脫離不了剛才的範圍

Adobe Photoshop 的濾鏡選項



有興趣的同學有空再慢慢自己去玩吧!

Review

- Intensity transform function
  - Contrast stretching and thresholding
  - Gamma correction
- Histogram and Equalization
- Spatial filtering
  - Low-pass filters
  - High-pass filters

生醫影像研究方法：
影像亮度轉換與空間濾波